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The Role of Real Estate in a Mixed-Asset Portfolio and the Impact of Illiquidity

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Abstract

Real estate ratios have increased in recent years. This article thus examines the diversification potential of real estate investments that German investors can achieve at a global scale. To this end, it analyzes how the illiquidity of some real estate investments or the illiquidity preference of an investor can bring about optimal investment ratios. Optimum allocation quotas for German investors with a wide range of mixed-asset allocations are examined. In addition to traditional optimizations, this article applies the three-fund theorem to include liquid and illiquid forms of real estate investment and to determine optimum allocation ratios. While real estate may be an essential component in a mixed-asset portfolio, it is not included in all optimal portfolios. An optimal portfolio also depends on the investment form, insofar as real estate vehicles are not always suitable for diversifying the portfolio risk or for improving the performance of a mixed-asset portfolio. Moreover, illiquid investment vehicles can often provide strong diversification benefits. The optimum allocations to real estate thus depend on the investor's illiquidity acceptance, even if allocation dominance has increased in recent years. While many studies have demonstrated the advantage that investors gain from adding certain real estate assets, such as those obtained by direct investments, this study goes further by examining the comparative advantage of different real estate investment forms within a variety of asset classes. New insights can thus be gained by considering investors' liquidity preferences within a given portfolio. One of these insights is that there is a trade-off between illiquidity and diversification potential. Another is that optimum portfolio allocations depend on illiquidity acceptance. These findings therefore also provide practical guidance not only to German investors with a global portfolio diversification but also to practitioners who add illiquid asset classes to their portfolio, to say nothing of the valuable field knowledge it offers to researchers in this field.

Keywords: Real estate, mixed-asset portfolio, real estate vehicles, illiquidity

1.0 INTRODUCTION

Economic crises, such as the one caused by the Covid-19 pandemic, have resulted in significant intermediate losses for some asset classes (Samson et al., 2020) and thus demonstrate the importance of a well-diversified portfolio (Kemper et al., 2012; Tai, 2018). For a long time, however, the classic investment spectrum of private and institutional investors included a combination of risky and fixed-interest securities. The risk appetite of the individual thus determined the weighting of their corresponding adjustments (Hudgins, 2012). Worth $228 trillion in 2016, real estate is the asset class with the highest value, while the combined value of all stocks,shares, and bonds is only $170 trillion (Savills World Research, 2017).

After the Global Financial Crisis of 2007-2008, the allocation to real estate as an alternative asset class began to rise. Motivations for investors to allocate to alternative assets were wide-ranging: from political uncertainties and low-interest rates to an increasing multi-asset allocation, to say nothing of the defensive character, higher returns, and the superior overall performance of real estate in comparison to other asset classes (Bals et al., 2016; Rock et al., 2019). These factors have led investors to regard real estate not only as an alternative asset class but also as an essential portfolio component (Söhnholz et al., 2010).

According to a recent study by Catella (2019), the average real estate ratio across all groups of German mixed-asset investors is around 9.8%, while the targeted ratio is 11-13%. However, the rates vary widely, depending on the class of investors. While life insurance companies have recently reported low ratios (around 4%), the ratios of pension funds are in the double-digits (some are even over 20%) (Rock et al., 2019), and foundations have even reached 26% in 2016 (Winkeljohann et al., 2016). Family offices already had a ratio of 22% in 2008 (Söhnholz et al., 2010) and were valued at 34% in 2014 (Schmitz, 2015). According to a recent survey by Ernst & Young Real Estate GmbH (2020), insurance companies constitute the most significant group of German institutional investors. Up from a 6% ratio in 2009, this group now accounts for a real estate ratio of 10.8%, thus signaling an upward trend.

The growing importance of real estate in portfolio allocations raises the question of the optimum real estate ratio in a mixed-asset portfolio. The literature review that follows discusses how scholars have tried to answer this question by examining investors' real estate allocations and the calculations they have made based on different financial portfolio models. Markowitz (1952), Grubel (1968), Levy and Sarnat (1970), and Solnik (1974) have approached the question from a globally-oriented portfolio framework, demonstrating that global allocation strategies have become more prevalent in practice.

2.0 LITERATURE REVIEW

Maurer and Sebastian (1999) and Hübner et al. (2004) have focused exclusively on the German real-estate stock market and found no significant diversification effects. However, Burns and Epley (1982), Mueller et al. (1994), Chandrashekaran (1999) and Jandura (2003), Ennis and Burik (1991) and Mull and Soenen (1997) have examined the US market along with the US-REITs in a global mixed-asset portfolio and found evidence for diversification. In a more recent study, Habbab et al. (2022) came to the same conclusion after analyzing UK, Australian, and US-Reits. Jandura (2003) investigated European stocks and observed the same dynamics as appear in US real-estate stocks. Likewise, Bond and Glascock (2006) and Newell and Marzuki (2016) came to the same conclusions after studying European investors. Unfortunately, there are studies of real-estate stocks and direct real-estate investments primarily for the US-Market. For example, Feldman (2003), Mueller and Mueller (2003), and Pagliari (2017) have demonstrated that both investment vehicles can be utilized within efficient portfolios. Corroborating this conclusion, Delfim and Hoesli (2019) have further shown that real-estate stocks are present in riskier portfolios, while direct investments appear more often in risk-averse portfolios. In the context of global allocation, Hoesli et al. (2004) have examined various national investors along with their domestic assets. They concluded that direct investments are almost always represented, but real estate shares are not represented at all. Examining the German market at the national level, Just and Maennig (2017) have concluded that direct real estate investment offers much potential for diversification (in conjunction with stocks and bonds) in relation to real-estate stocks.

Maurer and Stephan (1996), Hübner et al. (2004) and Maurer et al. (2004) studied German open-end real estate funds on a national level. Examining the same funds but within the context of global allocation, Haß et al. (2010) identified real diversification benefits from them. The latter also showed that German REITs are dominated by the open-end funds and thus gain no portfolio share. In contrast to Haß et al. (2010), Hübner et al. (2004) have demonstrated an increase in the efficiency of the portfolio for both forms: real-estate stocks were efficient in risky portfolios, while real-estate funds were efficient in risk-averse portfolios.

Regarding the optimal real-estate ratio, most of the extant literature concerns investors in foreign markets; however, the range of referred quotas for investors in the US market is between 5-45%. Ennis and Burik (1991), for instance, recommend 10-15% REITs; Feldmann (2003) up to 45% (39% of which should consist of direct investments and 15% of REITs); Pagliari (2017) suggests 10-15% (either REITs or direct investments depending on risk-attitude); Delfim and Hoesli (2019) propose 10-15%, allowing further alternative asset classes, such as hedge funds, commodities, and private equity. The ratio proposed by Hoesli et al. (2004) comes the closest to our estimations: 5-15% if the assets are unhedged and 15-25% if hedged (for real-estate stocks and direct investments). Haß et al. (2010) have proposed a similar ratio of 13-25% open-end real estate funds depending on investor type and risk-attitude. As regard to German institutional investors, Leser and Rudolf (2003) have proposed a real-estate ratio of 10-20%.

While most studies on this topic focus on US investors, and a few focus on European investors (including domestic assets), research on the world of German investment is scarce. In particular, there is room for more work on a mixed-asset, fully global allocation strategy using multiple real estate vehicles.

Furthermore, when they do investigate illiquid real estate, many scholars such as Markowitz use traditional financial models that, according to ideal capital market assumptions do not satisfy or rather infringe on the requirements of these models. Illiquid assets in particular do not fulfill these assumptions – for instance, the absence of transaction barriers and costs as well as unlimited divisibility. At least they do not indicate that they do consider this in any other appropriate way. These models are thus not capable of adequately accounting for characteristics of illiquid assets such as direct investments. This is because short-term returns of illiquid assets have a low information value and are thus not comparable to liquid asset returns (Söhnholz et al., 2010). Moreover, long-term returns would disadvantage the performance of the liquid portfolio, insofar as interim fluctuations are optimal. In this scenario, long-term development and liquidation should be possible at any time with the best possible result.

3.0 METHODOLOGY

3.1 Financial Models / Approaches Applied in Analyses

Different financial models are used to examine optimal real estate allocations and the changes within a mixed-asset portfolio that have come about in the last few years. Models are chosen depending on the characteristics of the real-estate vehicles' liquidity. In addition to traditional models, such as Markowitz optimization and Tobin separation, a three-fund-theorem is used to examine illiquid investments.

The reason for applying this model to illiquid real-estate assets is because traditional financial portfolio models do not adequately take illiquid assets into consideration. The risk measure volatility (variance or standard deviation) of traditional models is simply not suited to the study of illiquid data. Additionally, illiquidity is a limitation that is scarcely taken into account but has (Söhnholz et al., 2010). The Investment Property Forum (2012) has thus drawn attention to the complexity of capturing the risks of direct, real-estate investments and correlating them with other investments. Simple models, such as the classic mean-variance optimization, cannot account for such complexity. As a result, the diversification potential of real estate is often overestimated. Although a premium usually compensates for the

risks illiquidity, optimization should take into account illiquidity preferences. Otherwise, the results of optimizing a portfolio without determining an acceptable level of illiquid assets may not be appropriate for every investor. In other words, the investors' individual liquidity preferences can have an impact on the optimum allocation. Ang et al. (2011) have also noted the dependency between investors' liquidity preference and optimum allocations.

The three-fund-theorem applied in this study is based on the theoretical approach of Söhnholz et al. (2010), which is consistent with the theory of three different assets: 1) a risk-free; 2) a risky liquid; and 3) an illiquid risky asset. In order to determine the optimum illiquid portfolios, the three-fund-theorem is thus supplemented with the work of Tobin (1958) and Sharpe (1966). A large part of misestimates are caused by low-informative values of short-term data. For this reason, it is possible to mitigate the misestimation of classical portfolio parameters when including illiquid assets by using long-term returns. In any case, only long-term performance and correlations are relevant to investments with a long-term commitment. However, this paper examines illiquidity in its proper form, while at the same time also satisfying the demands of liquid portfolios – namely, the possibility to liquidate at any given point of time with the best result. This possibility makes it necessary not only to rely on long-term returns but also to use short-term returns for building the liquid portfolio. Söhnholz et al. (2010) suggest an independent optimization of risky liquid and risky illiquid portfolios. By contrast, we propose a dependent optimization of the illiquid portfolio to avoid unintentional and disadvantageous long-term co-movements and correlations.

The first step of the three-fund-theorem approach, therefore, is the two-fund-theorem. To this end, we determine efficient portfolios and the optimum (liquid) market portfolio respectively by examining which short-term returns fulfill the requirements of a liquid portfolio.

The second step is to determine the value of the fixed weightings of liquid assets by the share that the liquid portfolio has of the total portfolio. Different optimum portfolios with 0%, 20%, 40%, 60%, 60% and 100% maximum share of illiquid assets are determined. The sum of the weights is thereby reduced to the liquid portfolio share without changing the relations within the liquid part. The third step is to optimize illiquid-liquid-portfolios. This is done based on the quarterly-returns and quarterly variance-covariance-matrix for all asset classes and by optimizing the Sharpe ratio. By considering short-term data and movements, we ensure that the optimum illiquid-liquid portfolio satisfies the requirements of a liquid portfolio. The illiquid portfolio is thus deemed optimal due to the alignment of all components of the liquid portfolio by using long-term correlations that also ensure a desirable long-term development of the overall portfolio.

These empirical studies are generally with a global allocation strategy with unhedged/speculative in exchange-rate perspective. In addition to money market investments, however, real-estate funds also use currency hedges to a some degree. The effect of bonds without currency-risk has been analyzed in particular, as they usually serve to anchor risk and are therefore invested without such risk.

So long as there is a brisk stock exchange trading in them, open-end real estate public funds can be regarded as a liquid asset. There are two different kinds: a liquid version, which is held as a liquid asset and priced based on stock market trading and an illiquid version, which disregards exchange market prices and is based on the performance of the investment company's price of issue and redemption. This differentiation creates two completely different assets with different characteristics since the variant based on stock exchange prices has a higher volatility, correlation with the stock market, and is also considered liquid.

The study periods of the data presented in this paper deviate from the usual period (2006-2020) in the case of direct investments due to the availability of data. Furthermore, other periods were applied to verify stable results or to examine changes over time.

3.2 Data Used for Analyses

The data used in each analysis are based on the indices listed and explained below (see Table 1 for an overview of datasets). For liquid real-estate investments, we considered real-estate stocks (represented by NAREIT Global Index and MSCI ACWI Real Estate Index), US-mortgage-backed-securities (Bloomberg Barclays US Mortgage-Backed-Securities Index), and open-end real estate public funds. For the final item, we built an index based on exchange prices and distributions of these funds calculated with the internationally recognized BVI-method[1]. This index includes the 5 funds: (Union Investment) UniImmo Europa and UniImmo Global; (CommerzReal) Hausinvest; (Deka) Immobilien Global; and (DWS) Grundbesitz Global German funds. These funds were used to ensure that the correct currency effects were not distorted. Also, OEF-index, which measures the performance of many open-ended funds, were deliberately not used, as this included some funds that had to be liquidated because of problems that developed during the financial crisis. These are therefore considered not as representative for the future, insofar as the legal framework created since then would avoid further crises and thus the historical returns could be misleading.

A second index is calculated on the basis of the same funds and calculation method as the liquid OEF-Index. However, this excludes the possibility of liquidation by way of exchanges and is named illiquid OEF-Index. The investment companies' published redemption prices thus build on their valuation and, as a result, represent an illiquid alternative. In addition to these, the MSCI SFIX Index is taken into consideration, because it is generally considered a reliable proxy. Finally, the IPD dataset is used for a global and a Euro-countries index variant.

Apart from the aforementioned real-estate indices, the mixed-asset portfolio used for analysis always includes the following: the asset classes stocks (MSCI ACWI Index); investment-grade bonds (Bloomberg Bar-clays Global Aggregate Bond Indices and S&P Eurozone Investment Grade Bond Index); Commodities (Refinitiv/CoreCommodity CRB Index); and money market (DWS Vorsorge LC Fund).

Additional illiquid asset classes in some of the calculations also include: private equity (CA Global Private Equity Index) and hedge funds (HFRI Fund Weighted Composite Index). As a risk-free rate, an average of the EONIA EURIBOR was used.

Table 1 Overview of used data sets

Asset class	Representing index	Explanation
Real estate		
Real Estate stocks /REITS	NAREIT Global Index and MSCI ACWI Real Estate Index	Compositions and components differ between the two, basically MSCI Index was used, but the results were also checked with NAREIT Index where possible
US-MBS	Bloomberg Barclays US Mortgage-Backed-Securities Index	
Open-ended (public) real estate funds	Liquid OEF Index (self-calculated)	Based on exchange prices and distributions of these funds
	Illiquid OEF Index (self-calculated)	Based on the investment companies' published redemption prices which build on their valuation
Direct Investments	IPD Global Index	Includes all global countries covered by IPD/MSCI
	IPD Euro country Index	Includes only Euro countries
Other asset classes		
Stocks	MSCI ACWI Index	
Bonds	Bloomberg Barclays Global Aggregate Bond Indices S&P Eurozone Investment Grade Bond Index	
Commodities	Refinitiv/CoreCommodity CRB Index	
Money Market	DWS Vorsorge LC Fund	
Hedge Funds	HFRI Fund Weighted Composite Index	
Private Equity	CA Global Private Equity Index	

4.0 RESULTS OF ANALYSES FOR DIFFERENT REAL ESTATE VEHICLES WITH DIFFERENT APPROACHES

4.1 Empirical Analysis for Liquid Assets

First, the Markowitz and the Tobin models were applied to help in analyzing the role of real estate in a mixed-asset portfolio for vehicles that can be viewed as liquid assets (see Table 2 and 3). The following analyses are based on the performance data that resulted.

Table 2 Performance data for all asset classes with real estate funds, July 2006 - June 2020 (based on quarterly-returns)

	Stocks	RE Stocks	Bonds	Commodities	Money Market	US-MBS	Liquid OEF	Illiquid OEF	Hedge Funds	Private Equity	SFIX
Av. Return	8.65%	6.71%	4.99%	-3.31%	0.60%	5.87%	2.83%	2.98%	2.77%	13.25%	3.99%
Standard deviation	16.45%	19.69%	7.13%	20.33%	0.63%	10.11%	3.09%	0.68%	8.01%	10.40%	1.21%
Sharpe ratio	0.48	0.30	0.60	-0.20	-0.18	0.51	0.69	3.35	0.26	1.21	2.71

Table 3 Correlation matrix for all asset classes with real estate funds, July 2006 - June 2020 (based on quarterly-returns)

	Stocks	RE Stocks	Bonds	Commodities	Money Market	US-MBS	Liquid OEF	Illiquid OEF	Hegde Funds	Private Equity	SFIX
Stocks	1.00										
RE Stocks	0.86	1.00									
Bonds	-0.02	0.14	1.00								
Commodities	0.63	0.47	-0.12	1.00							
Money Market	0.18	0.20	-0.17	0.29	1.00						
US-MBS	-0.03	0.04	0.85	-0.08	-0.09	1.00					
Liquid OEF	0.34	0.28	0.11	0.36	0.27	0.02	1.00				
Illiquid OEF	-0.14	-0.16	0.05	0.05	0.67	0.09	0.40	1.00			
Hegde Funds	0.84	0.67	-0.38	0.71	0.35	-0.44	0.38	-0.03	1.00		
Private Equity	0.77	0.68	0.21	0.56	0.12	0.34	0.23	-0.12	0.52	1.00	
SFIX	-0.07	-0.08	-0.15	0.01	0.01	-0.09	0.02	0.24	-0.08	0.01	1.00
OFIX	-0.11	-0.17	-0.04	0.01	0.43	0.03	0.26	0.79	-0.04	-0.09	0.53

Figure 1 represents the possible risk-return combinations based on a Markowitz optimization for a portfolio that by default considers the following asset classes: stocks, bonds, commodities, and money market. It illustrates what impact the addition of the three different liquid real estate investment makes (real estate stocks, US-MBS, and the calculated liquid open-ended public fund index). The figure shows that real-estate stocks offer no diversification benefits in this context, since the line overlaps with the line for "no real-estate vehicles." Different analyses were carried out using the MSCI Global Real Estate as well as the NAREIT Global Index, which have different criteria. However, real estate stocks never received a relevant portfolio share. In contrast, MBS offers many diversification benefits particularly for the riskier portfolios, while the liquid OEF offers benefits for the low to medium-risk range.

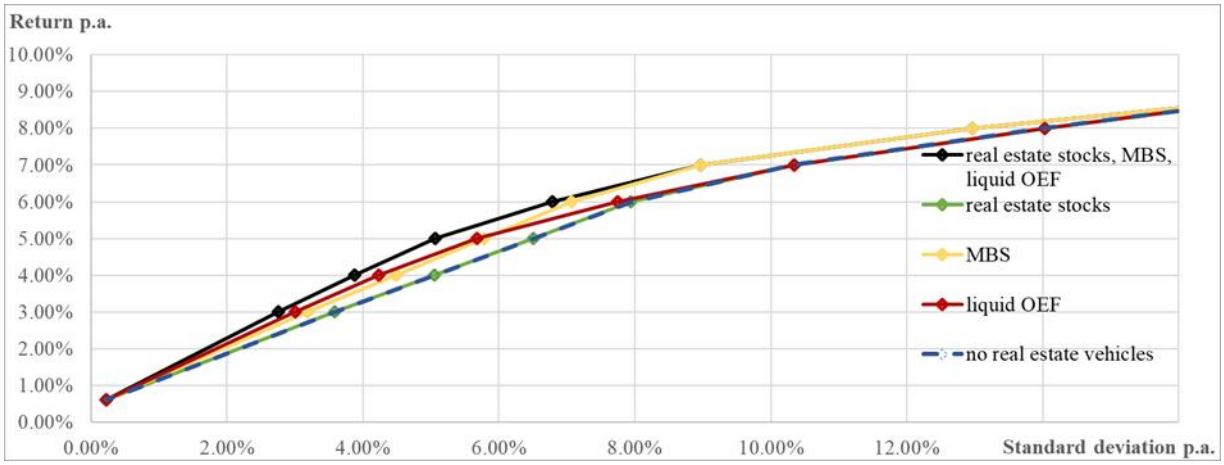
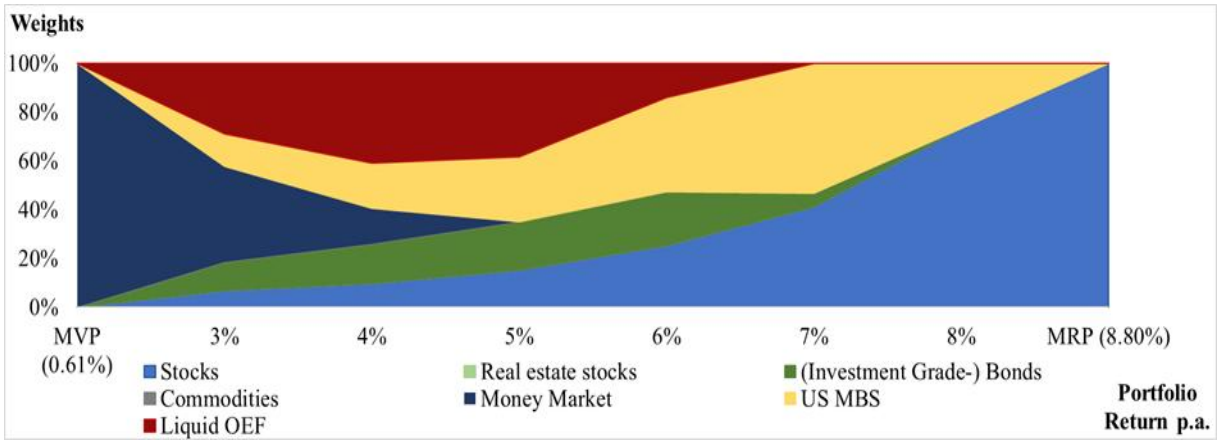


Figure 1 Diversification benefits by including (liquid) real estate vehicles, July 2006 - June 2020 (based on daily-returns)

On the other hand, Figure 2 shows the weights for the portfolio (which includes the same three liquid real-estate vehicles as in Figure 1) for the different return portfolios (from Maximum-Return-Portfolio to the Minimum-Variance-Portfolio with the lowest return). While Figure 1 illustrates the individual contribution measured as increasing of return or decreasing of volatility, Figure 2 illustrates the relatively role of those three different vehicles in mixed-asset portfolios with different return targets, measured by the individual share of the total portfolio.



MVP= Minimum-Variance-Portfolio; MRP=Maximum-Return-Portfolio

Figure 2 Weights of (liquid) real estate vehicles, July 2006 - June 2020 (based on daily-returns)

The Tobin approach uses the two-fund-theorem to determine the optimum allocations in a market portfolio along the efficient frontier. The average risk-free interest rate during the timeframe was 0.71% p.a. The analysis resulted in an optimum portfolio for the given timeframe with a return of 4.65% and a standard deviation of 4.61% annually, consisting of 22.3% MBS, 46.4% liquid OEFs, and 11.9% stocks. If liquid OEF's are not considered, as most institutional investors do not consider them, the optimum portfolio with an annual return of 6.27% and a standard deviation of 7.41% would have consisted of 40.5% MBS, 35.4% bonds, and 24.1% stocks. Real-estate stocks are thus not represented in the optimum portfolios. Therefore, US-MBS and the liquid OEFs were also present according to the Tobin approach, which also indicates the importance of these forms of investment.

Speculative bonds, however, are high in currency-risk, but Euro bonds are less volatile. When the latter are taken into consideration, the findings show that this inclusion has a significant impact on the allocation to open-end real estate (retail) funds. These bonds are thus

only included with a quite insignificant allocation in low-risk portfolios, suggesting that the open-end real estate funds, which exhibit low volatility, are dominated by Euro bonds in their previous role as risk anchor.

When US-MBSs are also considered as a hedged index the results show that the portfolio share of the Euro Bonds significantly increases, while the open-end (retail) real estate funds are not allocated. As a result, the share of US-MBS vanishes in all higher return portfolios. However, especially in the Sharpe portfolio, they are still present despite low returns on account of low correlations with the other asset classes, especially (negative correlations) with stocks.

4.2 Empirical Analysis for Illiquid Assets

In the subsequent part of this paper, the inclusion of illiquid real-estate assets is examined. First, a Markowitz approach is used for an initial assessment, even though the use of these traditional models is often regarded with suspicion due for the aforementioned reasons. The following analysis examines the diversification potential of direct real estate investments. Table 4 and 5 illustrate the performance data and correlations of all included asset classes.

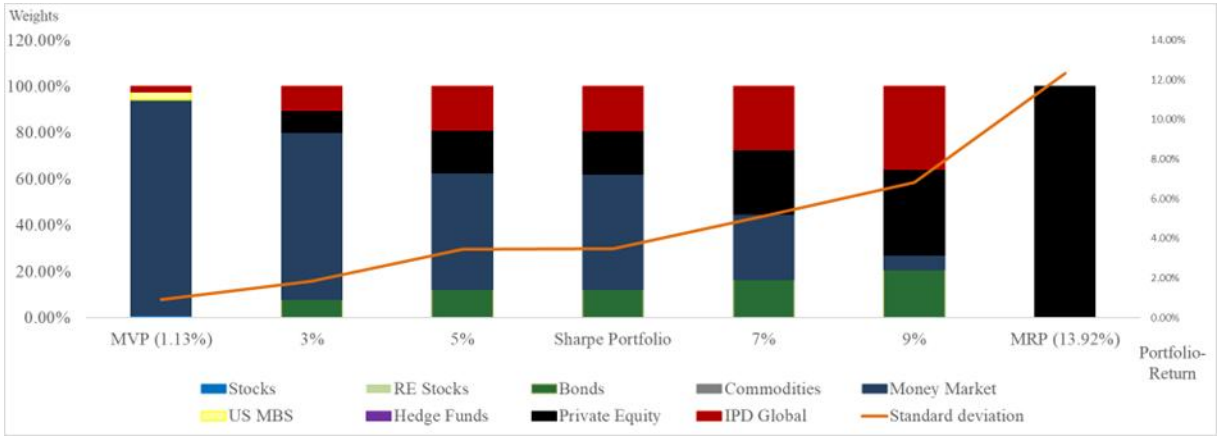
Table 4 Correlation matrix for all asset classes with real estate funds, July 2006 - June 2020 (based on quarterly-returns)

	Stocks	RE stocks	Bonds	EUR bonds	Commodities	Money Market	US-MBS	Hedge Funds	Private Equity	IPD Global	IPD Europe
Av. Return	9.27%	8.90%	4.33%	3.99%	-1.76%	0.70%	4.86%	3.53%	13.92%	8.10%	6.65%
Standard deviation	17.58%	21.92%	6.65%	3.62%	16.95%	1.04%	8.52%	9.12%	12.34	6.89%	3.67%
Sharpe ratio	0.48	0.37	0.53	0.88	-0.15	- 0.11	0.47	0.30	1.06	1.06	1.59

Table 5 Correlation matrix for all asset classes with IPD data for (illiquid) real estate investments, January 2006 - December 2019 (based on annually-returns)

	Stocks	RE Stocks	Bonds	EUR bonds	Commodities	Money Market	US-MBS	Hedge Funds	Private Equity	IPD Global	IPD Europe
Stocks	1.00										
RE Stocks	0.89	1.00									
Bonds	-0.04	0.10	1.00								
EUR bonds	0.17	0.28	0.47	1.00							
Commodities	0.74	0.59	0.18	0.09	1.00						
Money Market	-0.16	-0.07	-0.24	0.04	-0.03	1.00					
US-MBS	-0.07	0.07	0.96	0.45	0.06	-0.29	1.00				
Hedge Funds	0.87	0.75	-0.35	0.02	0.70	0.24	-0.40	1.00			
Private Equity	0.79	0.81	0.03	-0.13	0.55	-0.13	0.04	0.64	1.00		
IPD Global	0.13	0.28	0.57	-0.05	0.08	-0.26	0.60	-0.15	0.60	1.00	
IPD Europe	0.08	0.10	-0.30	-0.55	-0.06	0.00	-0.32	0.09	0.45	0.38	1.00

The results show significant diversification benefits and optimum allocations to direct real estate (IPD Global) with a share between the MVP and the MRP of 2.4-35.9%. In the IPD, the data excludes the use of debt, which is an additional common benefit that leverages the already outstanding performance of direct real-estate investment even further. On the other hand, no management costs are considered, which probably mitigates this effect. However, it should be noted that these characteristics of the data make it rather interesting from a theoretical point of view, since these returns reflect raw real-estate returns but have limited practical implications. Therefore, in the following section, real-estate fund returns are used as these represent real investable returns.



MVP= Minimum-Variance-Portfolio; MRP=Maximum-Return-Portfolio

Figure 3 Markowitz-optimization with IPD property data for direct (illiquid) real estate investments, January 2006 - December 2019 (based on annually-returns-calendar years)

Real estate is often used as a safe portfolio component – that is, as a risk anchor – when the exchange-rate risk is undesirable. Just as this applies to bonds, so we also examined the results of a portfolio-optimization, which contains Euro-dominated bonds and direct real estate investment within Euro countries instead of (unhedged) global direct investments and global bonds. The performance of direct real-estate in Europe demonstrates an outstanding risk-adjusted strategy that surpasses all other liquid and illiquid asset classes and also exhibits low correlation to stocks and bonds. The data was thus even more beneficial than the global variant, which resulted in a strong optimum allocation that ranged between 11.2% to 56.3% depending on the optimum portfolio. In comparison with the global variant, the place of real estate in the lowest-risk portfolio (MVP) is significant. Real estate and bonds, in fact, have the highest shares both in the Sharpe portfolio and in medium-risk portfolios. The standard deviations for all portfolios that include these two assets are significantly lower respectively the Sharpe ratios significantly higher. In this regard, Table 6 shows how portfolio performance is enhanced through the use of Euro Bonds and Euro-country direct investments and thus how to eliminate the threat of currency-risk.

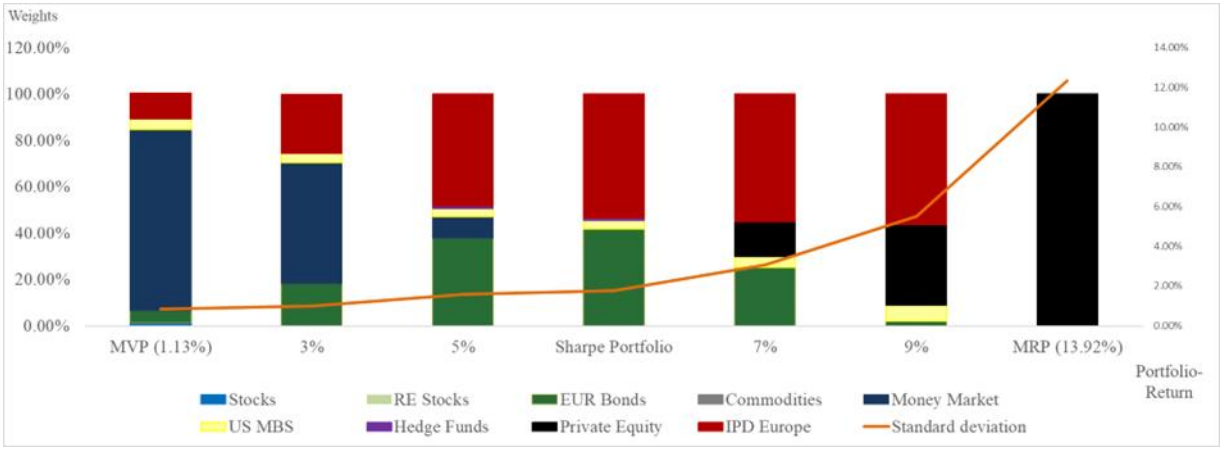


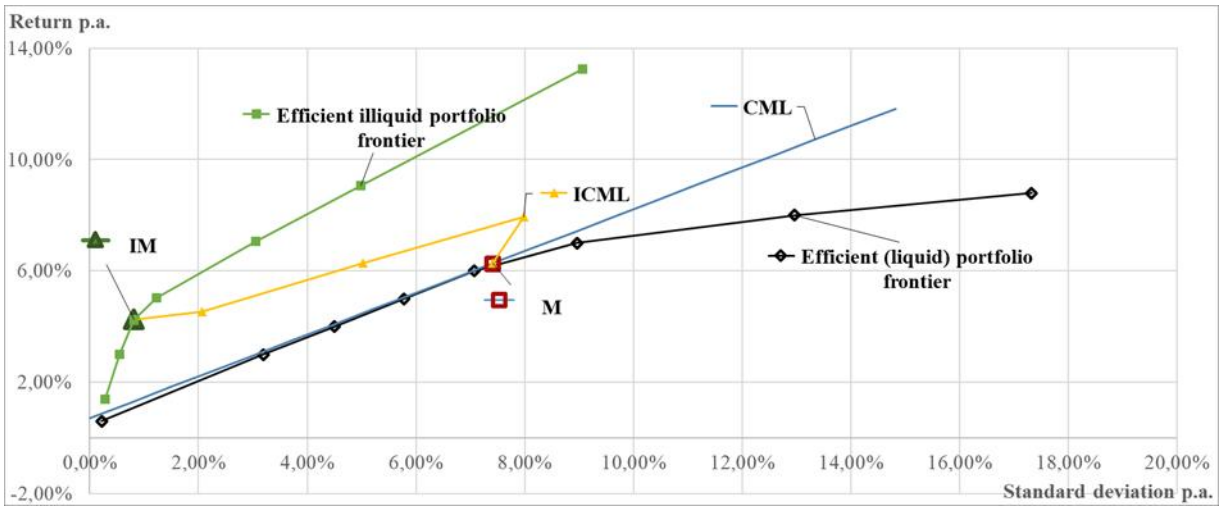
Figure 4 Markowitz-optimization with IPD property data for direct (illiquid) real estate investments with Euro Bonds and Euro country real estate, January 2006 - December 2019 (based on annually-returns-calendar years)

Table 6 Performance comparison between global variant (total currency risk) and use of Euro Bonds and Euro country direct investments only, January 2006 - December 2019 (based on annually-returns-calendar years)

		MVP	3%	5%	Sharpe Portfolio	7%	9%	MRP
Global variant	Return	1.13%	3.00%	5.00%	5.05%	7.00%	9.00%	13.92%
	Std.	0.91%	1.86%	3.46%	3.50%	5.13%	6.82%	12.34%
	Sharpe	0.34	1.17	1.21	1.21	1.21	1.20	1.06
Euro bonds and Euro RE	Return	1.79%	3.00%	5.00%	5.44%	7.00%	9.00%	13.92%
	Std.	0.85%	0.99%	1.59%	1.75%	3.06%	5.49%	12.34%
	Sharpe	1.15	2.21	2.63	2.64	2.02	1.49	1.06

These results already suggest a strong diversification benefit of illiquid real estate in a German national context. Whether this benefit is considered with or without currency-risk is immaterial, so long as the obstacles posed by traditional models are disregarded. Therefore, the following analysis (Fig. 4) illustrates the three-fund-theorem. Since this illustration is mainly for the purpose of showcasing the model, no investor-specific investments were chosen at this point. Only the following were thus included in the analysis: real-estate stocks and MBS for liquid real estate and the SFIX for illiquid. In addition to the traditional two-fund-theorem, we added an efficient and completely illiquid portfolio frontier as well as a mixed liquid-illiquid portfolio frontier called Illiquid Capital Market Line (ICML). The purpose of the efficient illiquid portfolio frontier is only to illustrate the possible risk-return combinations exclusively of illiquid portfolios (only real estate, hedge funds, and private equity). However, it is of no further relevance in this context. It should also be noted that the illiquidity of the individual asset is not measured as this is a further complex research topic. Distinctions were made only between liquid assets and illiquid assets. Under normal conditions, the former assets (liquid) can be bought and sold rather quickly without any constraints. Illiquid assets, on the other hand, cannot be bought and sold quickly. Portfolios were further differentiated between those that include and those that do not include a share of these illiquid assets. For the sake of simplicity, it is assumed for this illustration as well as for the following analysis that the whole liquid portfolio allocations consist of the liquid market portfolio so that no funds are allocated at the risk-free rate.

The ICML represents the possible risk-return portfolio combinations of the liquid-illiquid portfolios based on the three-fund-theorem approach. Because of the different illiquidity levels of the overall portfolio, the ICML does not become convex as better risk-return profiles are made available. It only becomes convex when higher illiquidity is accepted as a constraint. The reason why the ICML reverts back at a sharp angle around 8,00% is only because illiquidity actually represents a third dimension which is not visible in this graph.



CML=Capital Market Line; ICML=Illiquid Capital Market Line; M=Market Portfolio; IM=Illiquid Market Portfolio

Figure 4 Three-fund-theorem portfolio optimization, July 2006 - June 2020 (based on daily- and quarterly-returns)

Figure 5 below shows the weightings for portfolios with different illiquidity levels with 0%, 20%, 40%, 60%, 80%, and 100% illiquid portfolio components), which, according to the three-fund-optimization, lie on the ICML. The ratio that liquid investments, which will be optimized based on short-term returns, have to one another does not change, but only the overall share is reduced. This fact illustrates the once again the approach that has been taken.

The results show that for a portfolio illiquidity level of 20%, it is most beneficial to add private equity, while the share of illiquid real estate begins to rise with increasing illiquid portfolio share. From the 60% share that illiquid assets have in the overall portfolio, the addition of only illiquid real estate assets (SFIX-special funds at a certain level, illiquid open-ended public funds, and illiquid OEF) to the liquid portfolio is most beneficial. It can be observed that, from a certain illiquidity acceptance level onwards, the share of open-ended public real estate funds (Illiquid OEF) dominates the share of the special funds (SFIX). It can be observed that from a certain illiquidity acceptance level onward, the share of open-ended public real-estate funds (Illiquid OEF) dominates the share of the special funds (SFIX). Even though they can be considered less illiquid and have higher regulations toward liquidity reserves, the observation above is supported by stronger risk-adjusted performance (see full performance in Tab. 2). This, in turn, lowers the return, and thus a higher illiquidity premium may be expected. The figure below is based on a portfolio with speculative bonds high in currency-risk. However, the results remained constant despite different optimal liquid base portfolios according to various criteria: the influence of bonds with/without currency-risk during the various timeframes tested.

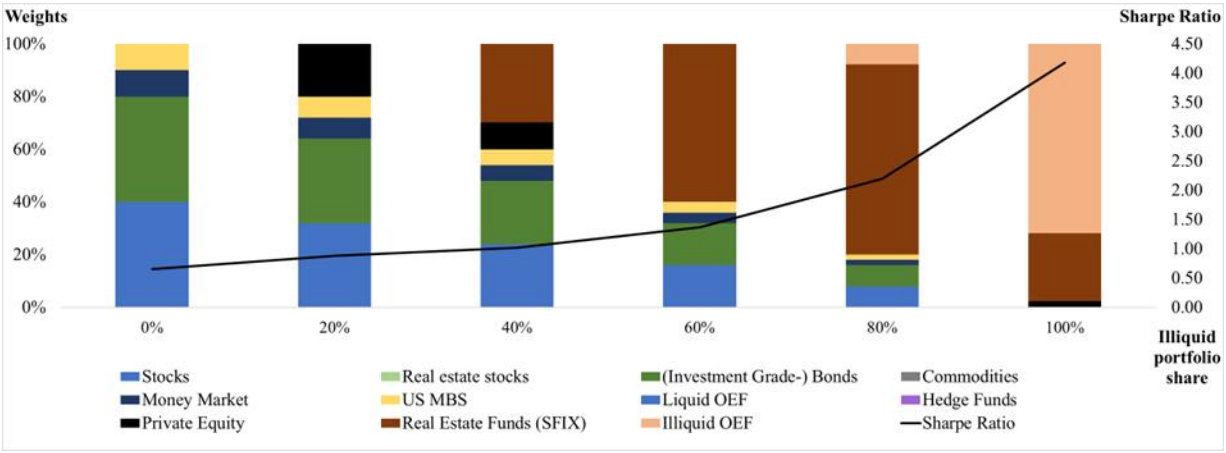


Figure 5 Optimal weights and Sharpe ratios for liquid-illiquid-portfolios with real estate funds, based on a reasonable liquid standard portfolio, July 2006 - June 2020 (based on daily- and quarterly-returns)

4.2 Changes in the Allocation to Illiquid Real Estate in the Most Recent Period

The purpose of the following analysis is to examine the optimum allocation for the different illiquidity levels during the period from July 2016 to June 2020. This period is deemed more representative of current conditions in the capital market. Table 6 and 7 show the performance data and correlation matrix for the following analysis.

Table 7 Performance data for all asset classes with real estate funds, Euro bonds, and hedged US-MBS, July 2016 - June 2020 (based on quarterly-returns)

	Stocks	RE Stocks	Bonds	Commodities	Money Market	US-MBS	Liquid OEF	Illiquid OEF	Hedge Funds	Private Equity	SFIX
Av. Return	11.20%	3.61%	2.09%	-4.81%	-0.41%	2.91%	1.43%	2.28%	1.46%	14.01%	6.71%
Standard deviation	17.38%	16.37%	4.84%	21.54%	0.21%	6.91%	4.96%	0.34%	8.96%	8.01%	0.50%
Sharpe ratio	0.67	0.22	0.43	-0.22	-21.53	-0.10	-0.82	5.47	0.19	1.05	11.57

Table 8 Correlation matrix for all asset classes with real estate funds, Euro bonds, July 2016 - June 2020 (based on quarterly-returns)

	Stocks	RE Stocks	Bonds	Commodities	Money Market	US-MBS	Liquid OEF	Illiquid OEF	Hedge Funds	Private Equity	SFIX
Stocks	1.00										
RE Stocks	0.87	1.00									
Bonds	0.09	0.36	1.00								
Commodities	0.88	0.82	0.06	1.00							
Money Market	0.90	0.74	-0.08	0.74	1.00						
US-MBS	-0.08	0.11	0.78	-0.03	-0.30	1.00					
Liquid OEF	0.76	0.75	0.00	0.80	0.78	-0.21	1.00				
Illiquid OEF	0.26	0.31	-0.16	0.56	0.26	-0.20	0.59	1.00			
Hedge Funds	0.93	0.77	-0.16	0.82	0.94	-0.41	0.78	0.34	1.00		
Private Equity	0.85	0.81	0.27	0.88	0.69	0.29	0.70	0.43	0.68	1.00	
SFIX	-0.32	-0.07	0.39	-0.12	-0.44	0.33	-0.07	0.25	-0.42	-0.13	1.00

During this period the illiquid OEF and the SFIX both show even higher Sharpe ratios than in the long-term analysis. This time, however, the SFIX performs better in comparison with the illiquid OEF, which can be explained by constant returns during these four years.

The following graph (Figure 6) represents the results for optimal allocations according to the three-fund-theorem for the last four years. In comparison with the long-term analysis, the allocation to real estate has slightly changed the illiquidity threshold, above which real estate is advantageous. It has slightly shifted to the left; however, depending on the composition of the liquid base portfolio, an even stronger shift can be noticed in some instances. The result is that real estate, in part, received all illiquid shares, thus increasing real estates' importance.

Furthermore, as the performance data suggested, the SFIX dominates illiquid OEF in recent years, so that, in the best case, only investment vehicles with special fund characteristics are secured. This effect was stable regardless of the composition and allocation of the liquid portfolio.

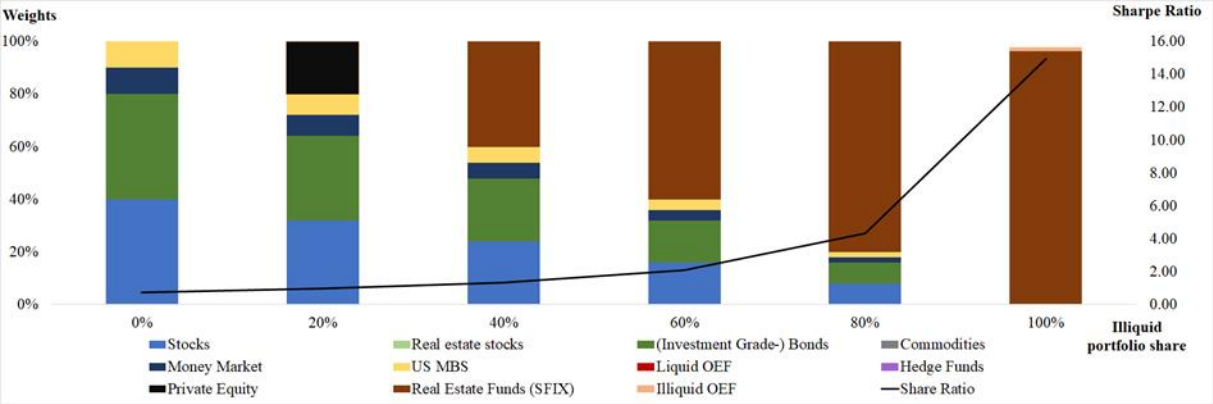


Figure 6 Optimal weights and Sharpe ratios for liquid-illiquid-portfolios with real estate funds, Euro bonds, and hedged US-MBS (both investor types), July 2016 - June 2020 (based on daily- and quarterly-returns)

5.0 DISCUSSION

The central research objective of this paper is to analyze the importance of real estate investments as a result of diversification effects. We thus set out to determine whether the rising allocation ratios of many investors are justified, especially from the perspective of a German Euro investor, by differentiating between investment vehicles, liquidity levels, and by exploring different approaches. Though the scholarly literature generally assigns some importance to real estate, few studies take into consideration German investors with an international allocation strategy. The results, whose empirical evidence is analyzed below, confirmed the manifold importance of real estate as an asset class in mixed-asset portfolios.

Concerning optimal real-estate ratios and forms of real-estate investment, much of the literature suggests that real estate stocks, open-end real-estate funds, and direct real-estate investments have an excellent potential for diversification. For the optimal real-estate quota, the literature suggests recommended range between 5-45%; however, those studies that are more in light with this paper propose 10-20% for German institutional investors (Leser & Rudolf, 2003), 5-15% for unhedged, and 15-25% for hedged international portfolios (Hoesli et al., 2004). For open-ended funds, in particular, a ratio of 13-25% (Haß et al., 2010) has been recommended. Based on our findings, it can be concluded that the answer is more complex than it is often presented in the literature. There are many different answers to the research question depending on the asset classes and investment forms included as well as the accepted portfolio illiquidity level.

The analysis of liquid real-estate assets in the mixed-asset portfolio context showed that MBS and, especially for retail investors, open-end real estate funds can offer much diversification potential, while real estate stocks are not present in any efficient portfolio.

The optimal allocation to liquid open-end real-estate funds and US-MBS are substantial when global assets are used. However, it was observed that when Euro bonds are included, they dominate the open-end real estate funds, and the optimal proportions of the funds almost completely disappears, while MBS continue to be a useful portfolio addition. Due to the focus on the US-market in an overall portfolio with a global orientation, the MBS has to be viewed in a differentiated manner, especially because of the influence of changing exchange rates. Nevertheless, the results clearly indicate a robust diversification potential and the advantage of combining US-MBS both with and without currency-hedge.

For real-estate stocks and REITs, many studies demonstrate that they have good diversification potential. However, the conclusion of this study is that real estate stocks are inefficient when actually applied. This finding seems to coincide with the results of Hoesli et al. (2004), who also examined real-estate stocks in a global mixed-asset portfolio and found that direct investments are always allocated, whereas real-estate stocks are insignificant. On the other hand, the studies which postulate the diversification benefits of real estate stocks often use only a pure REIT-Index, while the index used in this study also contains a certain proportion of other structured real-estate shares. However, the study of Bond and Glascock (2006) demonstrated that diversification potentials only exist in certain European countries. As a result, it seems that diversification effects only appear in some geographical portfolio orientations or within analyses carried out during a strong period of real estate stocks. Füss and Schindler (2011) suggested that for REITs (i.e., returns with a longer horizon than one year), long-term correlation to the real estate market increases and that the correlation to shares decreases. This effect would induce an obligation to a long-term capital commitment, which means that REITs cannot be considered within the liquid portfolio. If this effect were to be further investigated, REITs would have to be included in the optimization of the illiquid portfolio share.

When illiquidity and illiquidity-acceptance are taken into account, the determination of optimal real estate ratios appears to be much more complicated than some research suggests.

The following two tendencies can be observed from the results of including illiquid real estate in the analysis. The first is that illiquid real estate is a good portfolio diversifier with outstanding risk-adjusted performance. The same conclusion has been reached by Bond et al. (2007) and Delfim and Hoesli (2019), whose studies also included commodities, private equity, and hedge funds as further alternative

investments. The second is that there is no generally valid recommendation that can be made when different illiquid assets are included in the optimization. This is because optimal illiquid allocations depend on the total illiquidity acceptance of the investor for the overall portfolio. This results in illiquid real-estate assets that are not represented in every optimum mixed liquid-liquid portfolio. It is also reflected by real estate that is dominated by private equity in portfolios with less than 40% illiquidity. This result was very stable in long-term studies, even using different periods and different liquid portfolios.

The robust diversification benefits and thus portfolio shares of private equity are reflected by the renowned portfolios of the Harvard and Yale Foundations (see Figure 5 and 6 for our results up to a level of about 20%). The Yale portfolio in 2019, for instance, held a private equity share of 38% and a real-estate share of only 10% (Yale University, 2019). The Harvard portfolio in 2020 held a private equity share of 23% and a real-estate share of 7.1%, and they also intended to increase the private equity share even further (Harvard University, 2020).

Most of the scholarly literature, however, also derives optimum real estate allocations for illiquid vehicles, such as direct investments, by relying on traditional optimization models that treat liquid and illiquid assets as equal. Another tendency among researchers is to use approaches that optimize for longer investment horizons. These longer horizons partly meet the requirements of illiquid investments; however, they do not contain a liquid portfolio share. They are therefore unable to satisfy possible short-term consumption needs, since the investment horizon means that virtually all of them are considered illiquid for the duration of the investment horizon. Furthermore, the literature often does not include additional illiquid assets and thus it overestimates the diversification potential of real estate (Pagliari, 2017). For these reasons, studies on the practical application may in some cases be misleading. There is thus no universally applicable optimal ratio for illiquid real estate, insofar as both direct investments or real-estate funds and illiquid real estate investments were not always a reasonable addition. Ang et al. (2011) and Investment Property Forum (2012), whose results were discussed above, support the following two claims: 1) traditional models are not able to adequately account for illiquid assets and 2) liquidity preferences have an impact on the final optimal allocation. This also reinforces the necessity for an approach, like the one applied in this paper, to assess diversification effects more realistically and to better determine allocations by taking into account individual preferences.

However, it should be noted that the approach used in this study is not considered a fully developed methodology. Even if the results already provide interesting implications for practice, the methodology described is rather intended to point the way forward and stimulate similar research in the future. The authors are aware that some points still present difficulties or need to be optimized, such as the inevitable slope of the shares of the illiquid portfolio. This occurs when a sale is made from the liquid portfolio and turns out to be disadvantageous to the portfolio's overall performance. In addition, the use of optimal frequencies of returns has not been examined. As a result, annual returns could possibly be better than quarterly returns at determining the illiquid share, such that only their long-term development and co-movement are important.

One final consideration is that if the smoothing hypothesis is valid, the studies on all illiquid investment forms used in this paper should be conducted again with "de-smoothed data" or with transaction-based indices. Otherwise, the results could be quite different, reflecting increasing volatility and correlation with securities markets (see Booth & Marcato, 2004). However, it should be kept in mind that the smoothing hypothesis is not uncontroversial. For example, Lai and Wang (1998) criticize the fact that the attractive risk-return characteristics of real estate can be explained by illiquidity premiums and high information cost. These characteristics, therefore, do not indicate the existence of a smoothing effect. Furthermore, they show that real-estate returns can have even lower volatility than valuation-based indices (see also Webb et al., 1992). In addition, "de-smoothed data" sometimes also exhibits an even lower volatility (Bond & Hwang, 2007), which means that the terms "smoothing and "de-smoothing" begin to lose all meaning in this context. Regardless of whether the smoothing hypothesis is to be affirmed or not, some of the literature presented here also shows positive diversification effects and quite similar results for "de-smoothed" data. Such benefits can be seen in adding real estate to a mixed-asset portfolio (with different de-smoothing methods applied). Moreover, the investment ratios of market participants seem to reflect a similar assessment of the advantageousness of real-estate investments. For these reasons, the authors' opinion is that the fundamental findings of this paper, particularly in regard to illiquidity and the resulting dependence on optimal investment ratios, are not compromised in any way.

6.0 CONCLUSION

Real estate enables diversification potential that justifies investments in various forms. However, the individual investor's illiquidity acceptance has a decisive influence both on optimal investment ratios and on vehicles of investment.

Among liquid assets related to real estate, US-MBS seem to provide a sustainable diversification potential, while the advantageousness of illiquid real estate was found to vary in accordance with the illiquidity acceptance level, at least when private equity was considered. More specifically, illiquid real-estate investments can be a very good addition, because they dominate the other standard illiquid asset classes (hedge funds and private equity) above a certain illiquid portfolio share. This is because their risk-adjusted performance is robust; however, they are not represented in optimum portfolios below a certain illiquidity acceptance level or when portfolios allow a certain level of allocation to illiquid assets.

In comparison, most of the literature seems to overestimate as well as to generalize the diversification effects of (illiquid) real estate. This is mistaken, insofar as researchers have paid no or much less consideration of other illiquid asset classes or of the inter-dependence of individual illiquidity preferences (we have shown that illiquidity acceptance and optimum allocation are mutually dependent). A more sophisticated way to determine optimal ratios (as applied in this paper), therefore, seems appropriate. In response to the rising real-estate ratios of many investors in recent years, illiquid real estate may be able to play an important role. Of particular interest in recent years is the optimum share in portfolios that those with a low share of illiquid assets enjoy. Such a trend, however, cannot be confirmed for liquid real estate investments.

The findings of this study provide some interesting insights with practical implications relevant to the allocation of investment funds, portfolio managers and all investors, who must manage the complex task of including liquid and illiquid asset classes adequately in their portfolios.

While this study has provided some insights into liquid and illiquid real-estate assets in the context of a mixed-asset portfolio, there is still potential for further research in this area. First, it would be worthwhile to verify these effects by comparing them with transaction-based data. This would help to dispel possible objections to the smoothing hypothesis and its possible impact and also to extend the analysis to further real-estate related asset classes, such as the area of real estate debt. With regard to real-estate debt, which also accounts for a large proportion of the real-estate quotas of institutional investors, some studies have pointed to the good performance and diversification potential of European MBS (de Jong et al., 2015), since this study only focused on US-MBS and this could be a very interesting alternative due to the absence of currency risk. On the other hand, there is at the moment a generally strong trend towards real estate debt investments and in particular private real estate debt investments, which is reflected in a rapidly growing volume of debt funds, and capital inflows (EisnerAmper, 2019).

Note

1) BVI method was developed by the Bundesverband Investment and Asset Management e.V. (BVI). This method is used to calculate the performance of investment funds. The BVI method is an internationally recognized standard method; it enables a simple, comprehensible and exact calculation. The performance calculation according to the BVI method is based on the "time weighted rate of return" method. The performance of the investment is the percentage change between the invested assets at the beginning of the investment period and their value at the end of the investment period. Distributions are mathematically invested immediately in new fund units. This ensures the comparability of the performance of distributing and accumulating funds.

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