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A Modified UK Real Value/Short-Cut DCF Model for the Valuation of a Nigerian Sub-Leased Commercial Property

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Abstract

A UK real value/short-cut DCF model that addresses the valuation of geared profit rents generated by sub-let commercial investment properties especially in climes with nationalized land policy has been elusive. This study examines the modification of the extant UK real value/short-cut DCF model for the valuation of head-leased occupancy rights with fixed and non-revisable ground rents pursuant to the provisions of the Nigeria Land Use Act. The model is developed by blending the inputs of the modified rational model with that of the Crosby's real value/short-cut DCF model leading to a valuation model that appears simplified compared to the existing UK contemporary value models. The gearing effect for the hypothetical case of sub-let head-lease occupancy right in Nigeria is manifested as having a diminishing impact of fixed ground rent on profit rent calculation. For the valuation case study involving this hypothetical head-leased occupancy right in Nigeria, it was found that the modified UK real value/short-cut DCF model produced valuation that is identical to those churned out from the full DCF, modified rational, and Crosby's real value/short-cut DCF models. For the same hypothetical case study, less than 1% difference is observed between valuations from the all risks real yield (ARRY) model, which evolved in New Zealand and the valuation produced from the modified UK real value/short-cut DCF model, thereby validating all methods against each other. In consonance with the International Valuation Standards pertaining to income valuation approach, a sustained pedagogy of emerging real value models alongside the extant nominal valuation models is suggested as a way forward. Nevertheless, the modified UK real value/short-cut DCF model is recommended as an alternative to the extant variants of real value and explicit DCF techniques for the valuation of sub-let head-lease occupancy rights with fixed/non-revisable ground rents payable especially in climes with nationalized land policy.

Keywords: Geared profit rent, real value/short-cut DCF, right of occupancy, head-lessee's interest, property investment valuation

■1.0 INTRODUCTION

Land and property are synonymous to investments where the holder of the vested interest exercises the right to transfer possession to another party in return for periodic payments (FAO, 2017). An instance of this is a leasehold interest in landed property, which is a limited duration of tenure created by a title that confers possession and usufruct rights on a benefitting party (lessee) in exchange for a consideration otherwise called the ground rent made payable to the freeholder or landlord (Elias, 1971; Smith, 2007).

In Nigeria, the Land Use Act of 1978 had eroded all traces of allodial ownership of land and replaced it with statutory and customary rights of occupancy respectively, while conferring trusteeship status on state governors (for urban lands) and local government chairmen (for rural lands), and retaining ownership of Federal lands/landed properties in the Federal Government. These rights of occupancies conferred on desiring citizens are likened to leases owing to their terminable durations (Smith, 2007). An interpretation of the Land Use Act posits that the statutory or customary rights of occupancy conferred on individuals are terminable interests whose renewals are subject to the discretion of the state governor or local government chairman (Amokaye, 2011; Smith, 2007). In other words, at the expiry of the (building) lease, both the land and the building (unexhausted improvements on land) shall revert to the freeholder (Sayce et al., 2006), which in the Nigerian case is the trustee (a state governor), after which the individual can apply for renewal which shall however be subject to the discretion of the same trustee. Further interpretation of the Land Use Act indicates that the full reversion of land and building to the trustee is without any express provision for compensation unless an express revocation of right of occupancy in the public interest has been issued by the governor. Furthermore, the Act confers usufruct rights on its holders while allowing them to alienate by way of sale, assignment, or mortgage subject to the governor's consent by virtue of section 22 of the same Act (Amokaye, 2011; Otubu, 2018). Popular among the rationales for the grant of occupancy rights is the erection of buildings and other unexhausted improvements on land.

Onuoha et al. (2015) defines ground rent as a consideration which a tenant pays to the landlord for constructing and using a building upon the land owned by the landlord in such a way that the landlord remains the owner of the land while the tenant and his/her unexhausted improvements continue to occupy the land for a limited duration of time. In the context of this article, ground rent is a contract rent payable for the grant of right of occupancy. The charge and receipt of this rent in any given State of Nigeria varies across land

uses in a term construed as "*differential ground rents*". In other words, ground rents are not uniformly charged across land uses and specific locations of a state. For instance in Imo State, ground rents per square meter for industrial and commercial land uses exceed that charged for residential-, agricultural-, and private-led civic uses (Onuoha et al., 2015). By virtue of Section 5(d) of the Land Use Act 1978 (Nigeria), ground rents are subject to revision by the trustee who in this case is the state governor in respect of statutory grant. In practice, these ground rents may be revised at intervals of say 2, 3, or 5 years.

Gearing in the context of this study should not be construed as the use of borrowed fund to alter the pattern of equity returns. For the purpose of this study, Baum et al. (2014), Fraser (1993), McIntosh and Sykes (1985), and Sayce et al. (2006) define geared leaseholds as growth/inflation-proof investments that have profit rents exhibiting synchronous growth rates and revision period with sub-rents arising from a fixed or non-revisable head rent throughout the term of the lease. Within the context of occupancy rights in Nigeria, there is a possibility of properties with geared profit rents by virtue of the Land Use Act which empowered the governor (trustee) to waive in whole or part the conditions for the grant of right of occupancy (Otubu, 2018; Udo, 2003), including the reduction or freezing of ground rent payable in accordance with Sections 5(g), 17(1), and 17(2) of the same Act. For instance, when ground rent is fixed for an entire duration of a grant of occupancy right, a situation may arise where the sub-rent grows throughout the term of the grant while the impact of the ground rent in the determination of the profit rent diminishes considerably towards the end of the term.

From these analogies, it is appreciated that the valuation of investment properties cannot be disconnected from the working knowledge and application of laws pertaining to land tenure (Millington, 2000; Udo, 2003). In view of this, FAO (2017) proffered two vital recommendations – the first being that "valuation should be placed in the context of land policy" while the second is the need to surmount unique valuation challenges posed by customary and informal tenure rights through independent and objective valuations of these rights. While the question of unique and objective valuation of these rights revolves around the integrity of the valuation process, comprising the securing of brief, conduct of surveys and preparation of the valuation report, the placement of valuations within the context of land policy requires a review and restructure of existing valuation models that are in consonance with the land policy documents.

In consonance with the concept of time-value of money, the value (today) of an interest held in property is a function of the opportunity cost of investing equivalent series of increasing tranches of net income from the property at a market discount rate over the term (duration) of grant (interest). Unlike freeholds, the present value of leasehold annuities diminish as the duration of interest elapses (RICS, 1997b). In view of these facts, why would a valuer assume that a right of occupancy is automatically renewable upon expiry? What if the trustee exercises his/her powers and authority to deny the renewal of the right of occupancy and instead exercise eminent domain powers for overriding public interest? Similarly, Smith (2008) observed that no provision was made in the Nigerian Land Use Act for the application for renewal of occupancy right through a re-grant, so that insecurity of tenure is imminent. These adverse scenarios of land governance exacerbate insecurity of future series of increasing tranches of net income from a right of occupancy such that incompetence may ensue where terminable interest like rights of occupancy are valued using freehold valuation models.

Baum and Yu (1985a) had identified the inappropriateness of the conventional dual rate years purchase towards addressing the issue of gearing in leaseholds. In response, Baum et al. (2014) and Baum and Yu (1985b) had recommended the use of either the DCF or real value approaches in the valuation of geared leaseholds by splitting the valuation of head rent and occupation rent. In Nigeria and other countries with nationalized land policy, this approach appears to be most applicable to the valuation of rights of occupancy characterized by fixed ground rents and revisable sub-rents that are subject to growth over the term of grant.

Between the 1970s to mid-1990s, the UK contemporary value models that have been developed for the purpose of valuing geared leasehold profit rents include the explicit discounted cash flow [DCF] valuation technique (Baum & Yu, 1985b; Baum et al., 2014; Sayce et al., 2006), the modified rational model (Baum & Yu, 1985b); and the real value/short-cut DCF model (Baum & Yu, 1985b; Crosby, 1984). Besides these, there has been numerous studies leading to the development of an array of models for the valuation of investment properties ranging from freeholds to terminal interests (Jefferies, 2017b). Away from the UK to New Zealand, Jefferies, towards the end of the 20th century developed the generic 'real value' valuation model with adaptable Excel® template for the valuation of freehold and leasehold interests (Jefferies, 1997b). This model appears identical to the modified rational model except for the replacement of nominal discount rates and growth rates with real discount and capitalization rates. In the 21st century, Jefferies scaled up to develop an all risks real yield (ARRY) model which according to him is still work-in-progress (Jefferies, 2018b).

Notwithstanding the existence of other similar contemporary models for the valuation of terminable interests, a cursory look at the modified rational and real value/short-cut DCF models developed over the past decades indicates that there has been no similar study carried out to blend the inputs of the two models in such a way that would lead to the simplification of the Crosby's real value/short-cut DCF model into a modified valuation model for geared leasehold profit rent that still features the equated yield, all risks yield, inflation risk free yield and the unexpired term of the lease while remaining implicit about rental growth rate and/or the rent review period but taking up a structure that appears different from the original Crosby's real value/short-cut DCF model. In a comparative review of UK contemporary valuation models, Crosby (1996) and Crosby et al. (1997), reiterated these models offer valuers with identical techniques of value assessment such that it would be needless to identify any single model to be better than the others.

This study aims to develop a modified UK real value/short-cut DCF model for the valuation of geared leasehold profit rents. Objectives put forward to achieve this aim include blending the inputs of the modified rational model with that of the real value/short-cut DCF model; identifying the analytical surrogates of the modified rational model and the real value/short-cut DCF model from the blending process; developing an modified UK real value/short-cut DCF valuation model for geared leaseholds using the identified analytical surrogates; testing the modified UK real value/short-cut DCF model using a hypothetical case of a right of occupancy with geared profit rents; and validating the valuation of geared profit rents using extant explicit and real value models for leasehold investment valuations.

This article addresses the development of modified UK real value/short-cut DCF model for the valuation of occupancy rights or leaseholds where rent paid is fixed. The model is expected to simplify both the modified rational model and the short-cut DCF model by replacing complex structures of income multipliers with surrogates of investment valuation functions. The content of the ARRY model was not used in the blending process designed to obtain the input variables of the modified UK real value/short-cut DCF model; however, valuation from the ARRY model was compared with that ensuing from the modified UK real value/short-cut DCF model.

The modified UK real value/short-cut DCF valuation model developed in this article appears to be applicable to the valuation of head-leased occupancy rights created in countries that have a nationalized land policy where interest in land is vested in the head of a regional government to hold in trust for its citizens such that the citizens are only given usufruct rights designed to run for a limited duration. Other

conditions for the deployment of this modified UK real value/short-cut DCF valuation model are cases where a state governor exercises express and implied powers derived from extant land policy or land law to freeze or fully waive upward revision of ground rents if full compliance to such covenant might impose hardship upon the holder of right of occupancy. In Nigeria, the Land Use Act 1978 (Nigeria), in sections 5(g) and 17 provided for such a scenario.

This article is structured under nine sections. This section introduces the topic and the intent of the research. Section 2 is a literature review providing insight into the valuation of geared leasehold profit rent; section 3 is a review of the traditional UK models for valuing geared leasehold profit rents and features an array of the structure for growth-explicit and UK real value valuation models for geared leaseholds; section 4 is a discourse on the limitations of the UK traditional models; section 5 is an analytical discourse on the all risks real yield (ARRY) model of property investment valuation and the development of appropriate ARRY model that can address the valuation of geared leasehold profit rents in Nigeria; section 6 deals with the synthesis of existing models and concepts for the purpose of designing the modified UK real value/short-cut DCF for the valuation of geared leasehold profit rents in Nigeria; section 7 reports the testing and validation of the modified UK real value/short-cut DCF valuation model against other explicit DCF and UK real value models; while sections 8 and 9 captures the discussion of results and conclusion respectively. An area of further research was proffered in the concluding section.

■2.0 LITERATURE REVIEW

Attempt was made in this section to review extant concepts and models associated with the valuation of terminable investment properties, which is adaptable to head-leased occupancies in Nigeria.

2.1 The Concept of Geared Leasehold Profit Rent

For a head-leased interest, profit rent represents the difference between the sub-rent accruing to the head-lessee and the rent which the head-lessee pays to the landlord or freeholder (Baum et al., 2014; Blackledge, 2009; Fraser, 1993; Isaac, 2002; Sayce et al., 2006). It is not earned where rent received equals rent paid (Sayce et al., 2006), or during a case of over-renting where the sub-rent is less than the rent paid (Mackmin, 1995). In order to validate a profit rent, both the sub-rent and ground rent should be earned on the same terms, for instance the full repairing and insuring terms (Isaac, 2002); unless an adjustment is made in the valuation model to accommodate the differences in letting terms.

For the purpose of clarity, a *"quasi-lease"* or *"quasi-sublease"* was described by Metz (2018) as constituting occupancy rights in an estate. Therefore, if the grant of a right of occupancy in Nigeria creates a quasi-lease, it implies that the letting of any unexhausted improvement on the allocated parcel of land would equally create a quasi-sublease which generates profit rent to the holder of the occupancy right. Profit rent is used to determine the market value or worth of leasehold investment properties. Sayce et al. (2006) identified the two categories of profit rents to include the fixed profit rent; and the revisable profit rent. Fixed profit rent is the difference between non-revisable sub-rent and non-revisable head rent of identical duration. Sayce et al. (2006) likened the fixed profit rent to fixed income securities, while the revisable profit rent was defined as the difference between a revisable sub-rent and a fixed head rent.

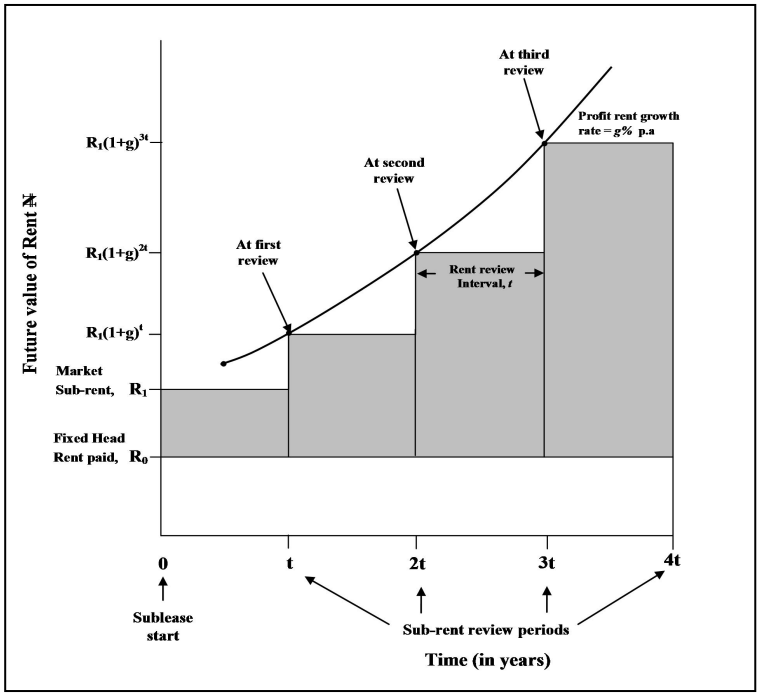


Figure 1 Geared leasehold profit rent
(Source: Modified from Baum et al. (2014) and Sayce et al. (2006))

The question now is – when is profit rent said to be geared? Existing texts pertaining to leasehold investment valuations have examined the concepts of geared leaseholds profit rents. Geared leaseholds otherwise categorized among the growth/inflation-proof investments (Baum et al., 2014) and indicated in Figure 1 are investment properties with profit rents (see shaded histogram) exhibiting growth rates and revision periods that are synchronous with that of the sub-rent, while head rent remains fixed or non-revisable throughout the term of the lease (Fraser, 1993; Sayce et al., 2006). In a similar definition, McIntosh and Sykes (1985) asserted that geared leaseholds are terminable interests in land that are characterized by upward review of rent payable to amounts that represent a proportion of the sub-rent received. Similarly, Sayce et al. (2006) reiterated that profit rents from geared leaseholds might represent a percentage of the revisable and growing sub-rent or leasehold market rent, while the ground rent remains constant throughout the term of the lease. Geared leasehold is however different from equity leaseholds which Fraser (1993) described as terminal investments characterized by regular upward revision of passing sub-rents to market sub-rents and the simultaneous revision of ground/head rent paid to the market ground/head rent thereby enabling a freeholder and leaseholder to share in the proceeds of the equity-sharing relationship and rental growth. According to RICS (1997b), gearing effects can be factored into profit rent calculation using explicit growth rates applied to leasehold market sub-rents at each rent review epoch. These insights are crucial to the explicit DCF valuation of geared leaseholds.

2.2 Insights into the Valuation of Geared Leasehold Profit Rent

According to RICS (1997b), the situations that leads to the creation of leasehold valuations include the assignment or subletting of a property at a rent in excess of rent actually paid to a freeholder described in the preceding section as profit rent. Conventional techniques for leasehold valuations where dual rates of interest are used to value profit rents have been criticized as illogical and failing to reflect leasehold market expectations (Baum, 1983; Baum & Crosby, 2008; Baum et al., 2014; Colam, 1984; Crosby, 1984; Sayce et al., 2006; Wood, 1986a) let alone the gearing effect of leasehold interests (Baum & Yu, 1985b). A full discourse of these criticisms is not intended in this article but the main thrust is how to use an alternative model to address the gearing effect of leasehold interests.

Baum and Yu (1985b) recommended the use of explicit DCF techniques for the valuation of geared leaseholds. About three decades after, Crosby and Henneberry (2016); Kucharska-Stasiak (2019), and Jefferies (2010) reported how UK valuers were reluctant to use explicit DCF models for market valuation. In the same vein, variants of explicit DCF valuation models have been resisted in favour of conventional techniques among valuers in Nigeria who derived a substantial proportion of their valuation tutelage and practice from the UK (Babawale, 2012; Idowu et al., 2012; Ogunba & Ajayi, 2007). Some critics anchor their reason for the rejection of the conventional model on the use of an (implicit) all risks yield, which according to them is mathematically inaccurate (Sayce et al., 2006). However, it would be observed that both the conventional and contemporary techniques share common input requirement of an all risks yield. Moreover, Brown and Matysiak (2000) affirmed that simple capitalization models are still based on the DCF concept. In other words, the use of all risks yield in valuations is rooted in the foundation of DCF so that the DCF in question complements the simple capitalization models being criticized by scholars in many quarters.

For a valuation model to align with the contemporary techniques, it is important that non-growth (nominal) incomes should be valued using equated yields, growth (real) incomes should be valued using real yields, and the intuitive adjustment of yields should be completely avoided. Instead of subtracting the rent paid from the sub-rent to determine the profit rent which would eventually be discounted to find the leasehold capital value, the contemporary approach to leasehold valuation entails finding the difference between discounted cash inflow [rent received] and the discounted cash outflow [ground rent or rent paid] (Baum & Yu, 1985b; Sayce et al., 2006). The rationale for this procedure is to handle the uniqueness of income pattern (Baum et al., 2014) and adequately address the problem of gearing in leaseholds (Baum & Yu, 1985b).

■3.0 REVIEW OF THE TRADITIONAL UK MODELS FOR VALUING GEARED LEASEHOLD PROFIT RENTS

3.1 Definition of Variables

The input symbols, notations, variables, and definitions associated with the traditional UK models for the valuation of geared leaseholds include:

PV = Present value or capital value of an unexpired interest in the geared leasehold.
R_1 = Rent received by the leaseholder (from sub-leases)
R_0 = Rent paid by the leaseholder (ground rent paid to landowner)
t = Leasehold rent review period (where synchronous with both R_1 and R_0 above)
g = Leasehold implied annual rental growth rate (of R_1 above)
e = Leasehold equated yield (EY required by head lessee and for R_1 and R_0 above)
i = Inflation risk free yield (IRFY as defined by Wood (1986b) and Crosby (1983))
k = All risks yield of leasehold income (ARY, for capitalizing both R_1 and R_0)
N = Unexpired term of leasehold interest (where synchronous with R_1 and R_0)

The subscripts 1 and 0 attached to R in the definition above was used to clarify rent received and rent paid by the leasehold respectively. With recourse to these symbols above, subsections 3.2 to 3.4 analyze the existing UK valuation models for geared leaseholds on the assumption that rents or cash flows are received at the end of the period (EOP) or in arrears.

3.2 The Full Explicit DCF Valuation Technique

This fundamental technique of investment valuation traces its origin to capital budgeting for the determination of net present value (NPV). The DCF technique is synonymous to the methodologies of income capitalization approaches to valuation (Brown & Matysiak, 2000; IVSC, 2003). Within the context of geared leasehold valuations, Sayce et al. (2006) demonstrated this technique to entail arranging each

tranche of cash flow according to its rent review interval, projecting the sub-rents using the implied rental growth rate and subtracting the (ground) rent paid from the projected sub-rent to arrive at profit rents for each tranche. Discounting this profit rent using the equated yield would amount to the discounted profit rent for each tranche. Besides the tabular layout, other approached to the DCF valuation technique include the conventional layout (Baum & Yu, 1985b), and the short-cut DCF technique (Baum & Crosby, 2008; Crosby et al., 1997).

In situations where it becomes necessary to shorten the length of the DCF or where the terminal sub-rent has a different review interval from that of the preceding tranche, Baum and Yu (1985b), Butler and Richmond (1990), and Crosby (1984) reiterated that equation 1 can be used to adjust the nominal cash flow to an equivalent value that would produce an acceptable result when discounted:

$$R_1^{'} = R_1 \times \frac{Y.P.\ for\ t_0\ years@e}{Y.P.\ for\ t_0\ years@i} \times \frac{Y.P.\ for\ t_1\ years@i}{Y.P.\ for\ t_1\ years@e} \tag{1}$$

Where R'_1= Amended equivalent of sub-rent in line with the new revision pattern where all other parameters remain as defined in the preceding section except that R_1 = sub-rent on the old review pattern, t_0 = the old rent review period, and t_1 = the new/amended rent review period. Equation 2 is an alternative to 1 where all the inputs remain as described and the use of the implied rental growth rate, g is preferable to the inflation risk free yield:

$$R_1^{'} = R_1 \times \frac{(1+e)^{t}0\ -1}{(1+e)^{t}0\ -(1+g)^{t}0} \times \frac{(1+e)^{t}1\ -(1+g)^{t}1}{(1+e)^{t}1\ -1} \tag{2}$$

It has been proven that the rent adjustment formulas (equations 1 and 2) are in-built in the short-cut DCF and real value techniques (Baum & Yu, 1985b; Crosby, 1984). However, when the tabular or conventional layout is used to value leaseholds with gearing potentials involving variation in rent review intervals, any of these equations could be used to adjust such rents before they are discounted at the appropriate rate of interest. Otherwise, there is no need for such adjustments where such situation is inexistent.

3.3 Modified Rational Model

Besides its application to the valuation of freehold interest, the "rational model" originally credited to McIntosh and Sykes (1983) was advocated for use in leasehold valuations as an alternative to the conventional leasehold models that had been criticized for yield adjustment and deceptive use of sinking fund element. According to McIntosh and Sykes (1983), the term "rational valuation model" was only chosen for convenience and that the ensuing model is a *"sophisticated and flexible modification of the equivalent yield model"* designed to value term rents and reversionary rents separately, account for the number of years to next rent revision, use a single rack-rented investment yield, and account for growth rate implied by the rack-rented investment yield [cap rate]. Following the discovery of the inability of the rational model to accurately value leasehold interest and account for the gearing effects, Baum and Yu (1985b) developed a modified rational value model as capitalization of incoming and outgoing rents expressed as:

$$Value\ = \frac{R_1}{k} - \frac{R_1(1+g)^N}{k(1+e)^N} - \frac{r}{e} - \frac{r}{e(1+e)^N} \tag{3}$$

So that when the aforementioned definitions in section 3.1 are applied to equation 3, it simplifies the valuation of terminating leasehold modified rational model as:

$$PV\ = \left(\frac{R_1}{k} - \frac{R_1(1+g)^N}{k(1+e)^N}\right) - R_0\left(\frac{1-(1+e)^{-N}}{e}\right) \tag{4}$$

In equation 4, the 1st term in bracket, values the sub-lessees' rental inflow, while the 2nd term in bracket, deducts the value of the head-lessee's rental outflow. With recourse to leasehold valuation case studies, Baum and Yu (1985b) reported how the original rational model developed by McIntosh and Sykes (1983) failed to produce valuations that reconciled with the real value and the DCF techniques respectively; while on the other hand, valuations produced by the modified rational model reconciled with that from the real value and the DCF techniques. In spite of their having modified the rational model to account for the gearing effect of leasehold interest and produce accurate valuations, Baum and Yu (1985b) still applauded the application of the explicit DCF and real value models over the rational model which they described as cumbersome.

3.4 Real Value/Short-Cut DCF Model

The real value/short-cut DCF model developed by Crosby (1983, 1984, 1986a, 1986b) also known as Crosby's 3-YPs model (Ataguba & Tinufa, 2015) is a synthesis of the equated yield model credited to Marshall (1976), and the real value model originally developed by Dr. Ernest Wood in his PhD thesis, which was subsequently published to support real value valuation using the inflation risk free yield (Wood, 1986a, 1986b). According to Baum et al. (2014), the real value/short-cut DCF model values the passing rent at the nominal yield but discounts the capital value of the market rent at the inflation risk free yield. Recalling Baum et al. (2014) as having advocated DCF or real value approaches in resolving the problem of valuing geared leasehold profit rents, Butler and Richmond (1990) demonstrated the application of the Crosby's real value/short-cut DCF model which is capable of valuing geared leasehold profit rent using equation 5 below:

$$PV = R_1\left(\frac{Y.P.in\ t\ years@e \times Y.P.in\ N\ years@i}{Y.P.in\ t\ years@i}\right) - R_0\left(\frac{1-(1+e)^{-N}}{e}\right) \tag{5}$$

It could be observed that equation 3 aligns with the characteristic of the real value/short-cut DCF described by Baum et al. (2014) in the sense that the sub-rent is valued using an income multiplier made up of a 3-in-1 years purchase formula (also known as the 3YPs model) designed to accommodate the inflation risk free yield, equated yield, rent review period, and unexpired term respectively. Unlike the modified rational model and the explicit DCF valuation techniques that require input of implied rental growth rate, the real value hybrid model [Equation 5] is implicit concerning growth rate and features the rent review period and inflation risk free yield instead.

■4.0 LIMITATIONS OF THE GENERIC UK MODELS

The generic UK investment property valuation models in the preceding sections are all founded on an annually in arrears, or end-of-period (EOP) rental payment timing basis, alongside the returns, interest rates and discount rates applicable to the financial markets. Similar assumption underlies the all risks yield (ARY), equated yield (EY), inflation free real yield (IFRY) and the basis of calculation of present value (PV) of discounted cash flows. These parameters were developed to value "reversionary freeholds" on a "term and reversion basis" but were found to "…produce distorted valuations of the term and reversion and can produce anomalous results" (RICS, 1997b, p. 16). This in turn implies that nominal growth (g) is annually on an EOP basis, whereas it actually occurs on a constant basis as illustrated in the market sub-rental exponential curve in Figure 1 and the projected leasehold profit rent in Figure 3.

The UK models in sections 3.2 to 3.4 above were developed for use by valuers during the era of manual hand calculations, and published valuation tables that were later superseded by the use of hand-held electronic calculators (Jefferies, 2009b, 2017b). Whilst actuaries have a highly complex mathematical method of valuing income on a constant growth basis, valuers do not, and have traditionally used the simple annually EOP method of calculations alongside the use of valuation tables because of their manual simplicity. This poses serious implications and should be acknowledged. For example, buyers of investment property pay out cash outflow on the day of purchasing an interest in property. Then they get cash inflows from rents (less any expense outflows) throughout the ensuing year, not waiting to get the whole rental income on the last day of the anniversary of acquisition.

Each tenant or lessee pays their rents – which could either be monthly as applicable to most western countries, quarterly for the UK institution leases, quarterly on a beginning-of-period (BOP) rental payment timing basis, or six-monthly during the year (RICS, 1997a, 1997b), such that their error effects on valuation results (French & Cooper, 2000). The UK models rarely allow for multiple terms and non-synchronous terms and condition across single properties with multiple leases and portfolios of separate properties except where explicit DCFs using spreadsheet programs are fully adjusted to address this phenomenon. Models must be able to be adapted to allow for different lease start and termination dates, rent reviews, expiries and renewals and for different outgoings, especially in multi-tenanted properties and also not on a universal assumption of a net-net-lease where the owner/investor has no outgoings being assumed to all be passed onto and paid by the tenants/sub-lessees.

Modern discounted cash flow (DCF) computer programs, especially spreadsheets were not available when these UK valuation models were created and came into use in the late 1980's and 1990's (Jefferies, 2017b), which can accurately deal with the realities of cash flow timing and frequency and the variable terms and condition of leases. These fully explicit DCF programs are used ubiquitously throughout the western business world, not these UK models, though some fully explicit multi-tenant programs are developed on a "term and reversion" basis (mainly in the UK).

Suffice it to say that these UK valuation models calculate the PV of delayed cash flows as at the date of valuation, implying that the purchaser remains out of pocket for a full year (12 months or 365 days) before getting all the rental cash flow on the anniversary day of the valuation date, or when valued for financial reporting purposes on the last day of the investor's financial year. Concurrently, all the expenses for the period are calculated as having accumulated and paid on the same last day of the year, which is not realistic in practice.

The all-risks element of these ARY, EY and IFRY yields deployed in the valuation are based on the analysis of comparable sales, which in the past traditional pre-computerization era, was the way it was done by both investors and valuers "following the market" evidence. In other words, the dictum of "as you analyze – so you value" implied a self-correcting error factor that was built into the practice and gave valuation accuracy within acceptable variance limits.

However, the basis of these UK valuation models has come under significant criticism in recent decades (Jefferies, 2010), and has led to the investment property market stakeholders and their advisers insisting on valuers relying on fully explicit DCF models for commercial valuations. This has relegated the old manually calculated models to purely academic exercise of model building, following an acknowledgement of their limitations in practice. For example, all risks real yield (ARRY) model to follow is theoretically based on the same foundation of an annual EOP generic model for mathematical purposes, and for the common purpose of comparison to these UK models, out of which it was developed. Notwithstanding, these limitations are drivers of adaptations for basic timing and frequency requirements, and for providing a flexible generic model that deals with reality associated with the varieties of modern lease market terms and conditions. Thus the ARRY model that follows is introduced in its fundamental annually EOP generic form and then is adapted, based on the published ARRY valuation models for freehold leased property (not head-leased property), and certainly not sub-leased ground-leased property, that is subject to practical real world limitations of its applications.

The model is then applied in this article to a valuation, as are the foregoing generic UK models, to a hypothetical Nigerian head-leased commercial investment property that is a sub-leased ground-leased property. This is, however, a quasi-lease in that it is not a formal ground lease – as in the UK and other commonwealth countries. Even the UK most sophisticated modified rational valuation model and the real value/short-cut DCF model were never envisaged to apply to such statutory rights of occupancy and tenures. This problem is also highlighted in other countries and involves unresolved valuation challenges as discussed by FAO (2017) pertaining to the valuation of such land tenure rights.

This case study is assumed to be built on a very long-term fixed quasi ground rental leased site with a shopping centre of rack-rented sub-tenants on net-net-leases commencing synchronously with the head-lease and for identical long-terminating sub-leases but with short-term rent reviews.

Their applications are therefore thwarted with considerable difficulty and in practice require adaptation of these traditional models. This also applies to trying to adapt the ARRY model, as to do so requires consideration of multi-levels of different property investment risk to reflect the hierarchy in different risks attached to the different interests being valued, where FH = the freeholder's risk or lessor in

the case of a normally leased property, that is of normal property investment risk; LH = the lessee's risk attached to being a tenant; HL = head-lessor's risk that is generally higher due to a combined risk of changes in both the head-rental and the sub-rental(s) cash flows and building operational management risks; and SL = the sub-lessee's highest risk attached to being a sub-tenant, though these interests are rarely valued, except in compensation cases.

However, the concern in this study is only with the head-ground lessor's HGL required real rate of return on net cash flow (sub-rental inflows less head-rent expense outflows) in valuing a head-ground lessor's leasehold interest. In practice if the head-rental is fixed and not subject to review, then the risk attached to that cash outflow will be less than that attached to the sub-rental inflows – but for the purpose of this following adaptation they are assumed to be the same. This simplification is implicit in the derivation of the market based analysis of the Nigerian economy and property market data in Section 7.1

■5.0 THE ALL RISKS REAL YIELD (ARRY) MODEL

The all risks real yield (ARRY) model Y_A where Y represents "yield" and subscript "$_A$" represents "all risks real", is an advancement from the real value traditional UK models of property investment valuation credited to Jefferies arising from array of seminal works on the subject matter on how to evolve a purely real value model. This was developed, however, primarily for an Australasian / North American valuation/appraisal professional readership and pedagogy, and follows their concepts and valuation terminology and methods.

Judging from the argument of Jefferies (2017b), none of the models reviewed above, including the modified UK real value/short-cut DCF model developed in this paper are strictly real value since they entail a combined use of nominal (equated) and real yields. The ARRY model exhibits strong synergy with all the other explicit DCF and real value models which qualifies its classification as a contemporary valuation model because it is a deviation from existing UK income capitalization models.

The origin of the real value concepts in the ARRY valuation model can be traced from models credited to Wood (1972, 1973) through Crosby's models in Crosby (1983, 1984) to the development of a generic net of growth models Jefferies (1997a). The timeline in the development of the ARRY model further include the application to the valuation lessor's and lessee's interests Jefferies (1997b); and a review of the history of real value models for investment property valuation including other income approaches (Jefferies, 2009a, 2009b, 2017b) that are comparable to the conventional and real value models. Other notable developments include arguments supporting the use of ARRY model in investment valuations (Jefferies, 2010); texts on the theory and practice of the fully developed ARRY model for investment property appraisals (Jefferies, 2016, 2017a, 2017c), a handbook explaining the automation of the ARRY valuation in spreadsheet templates and a set of practical case studies applying the ARRY model (Jefferies, 2018a, 2018b).

Jefferies (2016) defines the ARRY Y_A as *"..... the real internal rate of return that discounts the real values of the term to run and the real reversionary value(s) to the present real value (PRV), being the sale price or current market value."* In other words, ARRY is not a capitalisation rate, R but a real value discount rate (Jefferies, 2017b). With respect to the ARRY model, the market capital value of an interest in property investment equals the sum of the *"present real value of the contractual tranches of real income over the lease term to run and the PRV of the reversionary real value of capitalised future real rentals"* (Jefferies, 2010, 2016).

Contrary to the UK contemporary models, Jefferies (2017b) did not perceive the inflation risk free yield as a 'true yield' for the purpose of the ARRY model of property investment valuation because according to him, the ARRY capitalization rate, R_A^F for rent review terms of F years is based on all property investment risks excluding investors' assumed explicit expectation of inflation and real growth from the ARRY discount rate, Y_A, while the future real cash flows should be discounted using the ARRY, Y_A. The ARRY capitalisation formula utilises a quite different capitalisation rate formula to the UK models, being the present value PV factor applied to an annual in arrears annuity of 1 p.a. that increases at a constant growth rate at regular frequency of time intervals F, in perpetuity. This was first presented in Jefferies (2010), referencing to the formula's origins back to Rose (1979) in the UK, and in Australia (as documented in Worthington (1979, 1990), and being fully described and independently verified to be correct in Jefferies (2016, p. 381). This is a quite different formulation to the combined 3YPs approach of Wood (1972, 1973) and Crosby (1983, 1984), though gives the same result in a single formula.

The ARRY valuation model was developed for freehold properties with leases to tenants, – not head-leases – requiring a special adaptation that Jefferies did not develop. Adaptations for these applications were left for future research (Jefferies, 2016). Therefore, the model for this special head-lease-ground-leasehold on the assumption that rents are received in arrears, otherwise tagged end-of-period (EOP), has been expressed in equation 6. This entails duplicating this model in two-fold terms: the 1st term in square brackets values the sub-lessees' rental inflow, while the 2nd term in square brackets deducts the value of the head-lessee's rental outflow:

$$CMV = \left[\left(\frac{C_c}{R_A^F} - \frac{C_c}{R_A^F}(1+Y_A)^{-F}\right) + \left(\frac{C_c}{R_A^F}(1+Y_A)^{-F} - \frac{C_c}{R_A^F}(1+Y_A)^{-T}\right)\right] - \left[\frac{C_o}{R_A^T} - \frac{C_o}{R_A^T}(1+Y_A)^{-T}\right] \quad (6)$$

Where CMV = current market value (capital value); C_c = contract rent or leasehold sub-leases' market rental for M years' reviews if re-let at the valuation or sale date, C_o = passing leasehold head-lessees' rent, R_A^F = ARRY capitalisation rate of cash inflow, R_A^T = ARRY capitalisation rate of leasehold head-lessees' rent (cash outflow), Y_A = ARRY of cash inflow, F = review period of sub-rent, and T = unexpired terms of both the hypothetical synchronous head lease and sub-leases, by virtue of the standard definitions in Jefferies (2017c).

Prior to executing the valuation in the customized MS Excel template for ARRY valuation of investment properties, the variables defined above are inputted except that the values of R_A^F and R_A^T are calculated within the template as part of preliminary analysis of input variables for the actual valuation exercise. While the spreadsheet template assists to simplify the exercise, working from first principle would demand an understanding of the formulary. Where cash inflows are earned in arrears (EOP), the ARRY capitalisation rate for leasehold sub-leases' market rent with F review term frequency is expressed, where I_e = expected long-term monetary inflation, G_r = real expected long-term growth in rentals [where $(I_e + G_r) = G_O = g$ in Section 3.2, definitions as leasehold implied annual rental growth rate – see below] as:

$$R_A^F \Leftarrow (Y_A + I_e + G_r)\left[1 - \frac{(1+I_e+G_r)^F - 1}{(1+Y_A+I_e+G_r)^F - 1}\right] \quad (7)$$

On the other hand, $R_A{}^T$ being the ARRY capitalisation rate for cash outflow paid at the end of each year (EOP) over the unexpired term of the lease is expressed as:

$$R_A^T \Leftarrow (Y_A + I_e + G_r)\left[1 - \frac{(1+I_e+G_r)^T - 1}{(1+Y_A+I_e+G_r)^T - 1}\right] \quad (8)$$

Fortunately, in this hypothetical case study, the head-lease has no rent reviews thus T is both the term to run and to expiry. This would not occur in practice and is an important limitation of the model as presented here, and therefore not suitable as a generic model. Hence the previous requirement that a M years' reviews if re-let at the valuation or sale date was suggested above, as in the ARRY model so that these can be different and still function. Therefore, no other amendment to equation 6 is required in this hypothetical case study.

In equations 7 and 8, ⇐ indicates 'results from', and not an equality sign (=). With respect to the ARRY valuation models, capitalisation rates take into account the market's implied forecast real growth G_r where G is for rental growth and subscript $_r$ is for real, as distinct from the UK definition. The expected inflation rate is I_e where capital I is for inflation and subscript e is for expected, which should not be confused for the UK notation of equated yield. As noted above, the frequency of the rent review terms in years uses T for term to run (unexpired term), while F stands for the frequency of current market rent review. The ARRY capitalisation rates will usually be lower than the Y_A, where expected real growth rate G_r is positive, and the T<F where the valuation date is subsequent to the last review date and a shorter time to run and lower PV due to calculating to a higher $R_A{}^T$ than $R_A{}^F$. Also *vice versa* when the real growth rate G_r is negative which is most unusual in Western economies except may occur in recession or depression conditions or rampant CPI inflation or in a collapsing economy. In these situations, any real value model will produce valuations that are problematic, unstable and unreliable. The calculations of the all-risks real yield capitalisation rates are based on the overall total yield Y_O, where the subscript capital letter $_O$ is for overall. So that $Y_O = (Y_A+I_e+G_r)$. These separate and explicit components replace the over-all required yield Y_O and the total growth G_O is replaced by (I_e+G_r) in the standard ARRY capitalisation formula, giving greater transparency to the effects these separate market data inputs contribute to the valuation. It should be noted that the ARRY model is based on a per rental payment period overall yield Y_O, which is a summation the three components Y_A, I_e, and G_r. Hence, Y_A is derived from market evidence as $Y_A = (Y_O–I_e–G_r)$, contrary to the UK models' approach of compounding interest rates. Where there is no growth, i.e. $G_r = 0$; then the capitalisation rate equals the discount rate, so that $R_A{}^F = Y_A$. This is the result of a 'steady state' assumption where current nominal and real values are assumed to be fixed in perpetuity, a phenomenon which rarely exists in property markets. However, the application of this model to the Nigerian case study in this article is a simplified valuation of interest held under a terminating ground-lease situation.

Equation 6 above conforms to the ARRY valuation template in MS Excel as developed by Jefferies (2018a). While this article does not intend to dilute the very essence of the ARRY valuation model in equation 6, it is still possible to simplify it for the purpose of this study through equations 9 to 12 while retaining the variables defined above:

$$CMV = \left[\frac{C_c}{R_A^F} - \frac{C_c}{R_A^F}(1+Y_A)^{-F} + \frac{C_c}{R_A^F}(1+Y_A)^{-F} - \frac{C_c}{R_A^F}(1+Y_A)^{-T}\right] - \left[\frac{C_o}{R_A^T} - \frac{C_o}{R_A^T}(1+Y_A)^{-T}\right] \quad (9)$$

After collecting like terms and factorizing the common multiples, equation 10 ensues as

$$CMV = \frac{C_c}{R_A^F}\left(1 - \cancel{(1+Y_A)^{-F}} + \cancel{(1+Y_A)^{-F}} - (1+Y_A)^{-T}\right) - \frac{C_0}{R_A^T}\left(1 - (1+Y_A)^{-T}\right) \quad (10)$$

$$CMV = \frac{C_c}{R_A^F}\left(1 - (1+Y_A)^{-T}\right) - \frac{C_0}{R_A^T}\left(1 - (1+Y_A)^{-T}\right) \quad (11)$$

Further factorizing and collection of like terms would amount to:

$$CMV = \left(\frac{C_c}{R_A^F} - \frac{C_0}{R_A^T}\right)\left(1 - (1+Y_A)^{-T}\right) \quad (12)$$

While it is possible that equations 6 and 11 would yield similar results for the ARRY valuation of the property investment in focus, equation 11 does not feature that expression $(1+Y_A)^{-F}$ being the present value of cash inflow over the review period of sub-rent. In essence, equation 12 might appeal to readers who intend to obtain the value of interest in the property from first principle using handheld calculators without recourse to the spreadsheet template. For the purpose of this study, similar ARRY valuations shall ensue from the deployment of equations 6 and 12 respectively.

At this juncture, an attempt was made in the succeeding section to modify the existing UK real value model for the purpose of valuing sub-leased commercial property with gearing potentials within the Nigerian context.

■6.0 MODEL DESIGN USING A SYNTHESIS OF EXISTING MODELS AND CONCEPTS

The modified UK real value/short-cut DCF valuation model developed in this article is a blend of real value parameters with the modified rational model. The model redesign process is anchored on the assumption of the receipt and payment of ordinary annuities or cash flows received and paid at the end of the year (EOP).

6.1 Relationship between Nominal and Real Yields

Brown and Matysiak (2000) stated that the relationship between the nominal- and real yield is guided by the equation:

$$(1+e)=(1+g)(1+i) \tag{13}$$

So that the implied growth rate of leasehold income, g can be derived as:

$$g=\frac{(1+e)}{(1+i)}-1 \tag{14}$$

On the condition that $e > g$, the inflation risk free yield, i in the UK investment valuation context can be derived as:

$$i=\frac{(1+e)}{(1+g)}-1 \tag{15}$$

6.2 Model Development

Where all the following variables retain their original meaning as mentioned in section 3.1, the modified rational model developed by Baum and Yu (1985b) for the valuation of leaseholds with gearing potential can be recalled from equation 4 as:

$$PV=\frac{R_1}{k}\left(1-\frac{(1+g)^N}{(1+e)^N}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right)$$

Hence, an expansion of equation 4 becomes:

$$PV=\frac{R_1}{k}\left(\frac{(1+e)^N-(1+g)^N}{(1+e)^N}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right) \tag{16}$$

After inspecting equation 16, it was observed that "$\frac{(1+e)}{(1+i)}-1$" in equation 14 could be substituted for g as follows:

$$PV=\frac{R_1}{k}\left(\left((1+e)^N-\left(\cancel{1}+\left(\frac{(1+e)}{(1+i)}-\cancel{1}\right)\right)^N\right)(1+e)^{-N}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right) \tag{17}$$

Equation 17 was further simplified as:

$$PV=\frac{R_1}{k}\left(\left((1+e)^N-\frac{(1+e)^N}{(1+i)^N}\right)(1+e)^{-N}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right) \tag{18}$$

$$PV=\frac{R_1}{k}\left(\frac{(1+e)^N(1+i)^N-(1+e)^N}{(1+i)^N}\times\frac{1}{(1+e)^N}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right) \tag{19}$$

$$PV=\frac{R_1}{k}\left(\frac{\cancel{(1+e)^N}\left((1+i)^N-1\right)}{(1+i)^N}\times\frac{1}{\cancel{(1+e)^N}}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right) \tag{20}$$

Therefore, the modified UK real value/short-cut DCF model that can be used to value geared leasehold profit rent is stated as:

$$PV=\frac{R_1}{k}\left(\frac{\left((1+i)^N-1\right)}{(1+i)^N}\right)-\frac{R_0}{e}\left(1-\frac{1}{(1+e)^N}\right) \tag{21}$$

By factorizing the income multipliers, equation 21 could be written alternatively as:

$$PV=R_1\left(\frac{\left((1+i)^N-1\right)}{k(1+i)^N}\right)-R_0\left(\frac{1-(1+e)^{-N}}{e}\right) \tag{22}$$

With respect to equations 21 and 22, the implied rental growth rate, g has been concealed by an alternative relationship involving the inflation risk free yield. The modified UK real value/short-cut DCF model attempts to avail an alternative mathematical expression for the income multiplier for rent received using parameters that are attributable to the real value concept while retaining the valuation of rent paid, R_0 using the equated yield on the premise that capitalization rate for non-revisable and/or non-growth incomes remains the nominal rate of interest (Baum & MacGregor, 1992; Ifediora, 2005); in other words, $k = e$ when $g = 0$.

6.3 Array of the Structure for Growth-Explicit and Real Value Valuation Models for Geared Leaseholds

While the structure of the full explicit DCF valuation technique was deliberately omitted in this section and reserved for succeeding section, the valuation templates in Exhibits 1 to 5 have been prepared based on the convention understood by property investment valuers with experience of the UK models, while availing a catch-up readership for those with experience of the Australasian and North American markets. For all the valuation templates in exhibits 1 to 5, the capital value of the leasehold interest equals the difference between capital value of rent received and the capital value of rent paid. Across these templates, similarity is further exhibited in the structure of valuations of the fixed head (ground) rent paid, except for the ARRY model in Exhibit 5 which purely indicates the use of real value parameters.

Exhibit 1 features the template for the UK modified rational model. The income multiplier for rent received is computed in three stages. First is the calculation of the difference between the Amount of ₦1 in N years at e and g respectively. This differential in the future value of ₦1 is multiplied by the PV of ₦1 in N years at the equated yield to arrive at a factor which reduces the YP. in perpetuity at k to an equivalent Years purchase for terminal investments. This template affirms that leaseholds are derivative interests from freeholds.

Exhibit 1 Valuation template for the modified UK rational model based on Equation 4

					₦	₦
	Rent received				XXX	
	Y.P. in Perp. @ k			XXX		
	Amount of ₦1 in N years @ e	XXX				
Less	Amount of ₦1 in N years @ g	XXX	XXX			
	P.V. of ₦1 in N years @ e		XXX	XXX	XXX	XXX
Less	Rent paid				XXX	
	Y.P. for N years @ e				XXX	XXX
	Capital value of leasehold interest					XXX

Exhibit 2 features the template for the UK real value/short-cut DCF valuation model credited to Crosby (1983, 1984). The template contains the capitalization of rent received using the 3-YPs formula for leaseholds by virtue of the incorporation of N which is the unexpired term of the lease alongside the rent review period, equated yield and the inflation-risk free yield. The valuation of ground rent paid is identical to that found in the modified rational model in Exhibit 1.

Exhibit 2 Valuation template for the UK real value/short-cut DCF model based on Equation 5

			₦	₦
	Rent received		XXX	
	$\frac{Y.P.in\ t\ years\ @\ e \times Y.P.in\ N\ years\ @\ i}{Y.P.in\ t\ years\ @\ i}$		XXX	XXX
Less	Rent paid		XXX	
	Y.P. for N years @ e		XXX	XXX
	Capital value of leasehold interest			XXX

Exhibit 3 features the template for the modified UK real value/short-cut DCF model developed in this article for the valuation of geared leasehold profit rents. Although the valuation framework for sub-rent in Exhibit 3 is different from those in Exhibits 1 and 2, they are mathematically related in the sense that the product of the future value differential involving e and g and the PV of ₦1 function at the equated yield in the modified UK rational model was replaced with the calculation of the product of the compound interest on ₦1 in N years at the inflation risk free yield, and the PV of ₦1 for the term of the lease at the inflation risk free yield in the modified UK real value/short-cut DCF model in Exhibit 3.

Exhibit 3 Valuation template for the modified UK real value/short-cut DCF model based on Equation 22

					₦	₦
	Rent received				XXX	
	Y.P. in Perp. @ k			XXX		
	Compound interest on ₦1 in N years @ i.....	XXX				
	P.V. of ₦1 in N years @ i		XXX	XXX	XXX	XXX
Less	Rent paid				XXX	
	Y.P. for N years @ e				XXX	XXX
	Capital value of leasehold interest					XXX

Exhibit 4 features the template for the UK equivalent yield valuation of property investments. In the UK context, the equivalent yield is a single discount rate that is used to capitalize term and reversionary cash inflows of an investment property (Wyatt, 2013). This single

discount rate purports to reflect the entire attributes of an investment property comprising cash flow changes, and value at redemption (Ifediora, 2005). The equivalent yield valuation model for non-reversionary leasehold investments as presented in this study entails finding the profit rent at the end of the year being the difference between rent received and head rent paid at the end of the year and then capitalizing the said profit rent at the "single" equivalent rate. The model for the template in exhibit 4 is expressed as:

$$PV = R_1 - R_o\left(\frac{(1+y)^n - 1}{y(1+y)}\right) \tag{23}$$

Where PV = capital value of interest in the property, R_1 = sub-rent received (cash inflow), R_o = ground rent paid, y = equivalent yield, and n = unexpired term of the leasehold property.

Exhibit 4 Valuation template for the UK conventional equivalent yield model based on Equation 23

		₦
	Rent received	XXX
Less	Ground rent paid	XXX
	Profit rent	XXX
	Y.P. for *n* years @ *y*	XXX
	Capital value of leasehold interest	XXX

Among the advantages of the UK equivalent yield valuation technique is that it avails valuers with objective analysis of transactions (Baum & Crosby, 2008; Sayce et al., 2006) thereby making it the only conventional technique that can bridge the gap between growth implicit and growth explicit DCF valuation of reversionary property investments (Brown & Matysiak, 2000).

Exhibit 5 Extract from the MS Excel® template for the ARRY model of property investment valuation

Tenancy 1		Cell Definitions
Key Common data from SUMMARY Sheet:		
Valuation (or Sale) date: *Vdate*	00/01/1900	VDate
Expected (*nominal*) inflation rate: I_e	0.00000% p.a.	Ie
Real growth rate: G_r	0.00000% p.a.	Gr
Nominal value growth rate: $G_o = I_e + G_r$	0.00000% p.a.	Go
All-risks real rield (ARRY) rate: Y_A	0.00000% p.a.	YA
Over-all required *nominal* yield (disc. rate): $(Y_A + I_e + G_r) = Y_o$	0.00000% p.a.	Yo
CONTRACT LEASE DATES (Existing contract lease or assumed leasing terms if initial vacancy allowed).		
Lease commencement date: *Comm*		Comm
Lease expiry date: *ExpDate*		ExpDate
Term in years: *Term*	0.00 years	Term
Rental commencement date, (i.e. last rent review date if not *Comm* above), or after initial rent-free period if any: *RentComm*		RentComm
Initial rent-free payment periods (if any unexpired): *IRF*	Not applicable	IRF
Pre initial leasing refurbishment, free lessee's fit out or other incentive cost -$: *Contr*		Contr
Initial vacancy, in whole rental payment periods, to lease commencement (if applicable and unexpired): V_i	Not applicable	Vi
CONTRACT RENTAL DATA (Existing contract lease or assumed leasing terms if initial vacancy allowed).		
Premises annual rental: *PRent*		PRent
Car parks annual rental: *CpRent*		CpRent
Total annual contract rental: *Co*	$ -	Co
Monthly contract rental: *Co_pcm*	$0 p.c.m.	Co_pcm
Rental and OPEX payments frequency per annum: *P*		P
Rental payments: in advance (BOP) = 1; in arrears (EOP) = 0: *pay*		pay
Rental review frequency, or fixed rental term, during contract lease term - in years: *F*		F
IF prescribed rental - nominal annual escalation rate for rental: *Esc*		Esc
IF prescribed rental - current escalated rental if reviewed $p.a. : *CEsc*	Not applicable	Cesc
Ratchet clause?: *Rcht*		Rcht

YELLOW highlighted data input cells are unlocked and have validation boxes with data entry message. If invalid data is entered an *Error Alert* is shown. Tab ⇥ to next input cell.

For clearest viewing and for data inputting select Columns A:B; on menu View Tab, Click on "Zoom to Selection"

Presented in Exhibit 5 is an extract from the MS Excel® template deployed in the ARRY model of property investment valuation (Jefferies, 2018b). Appropriate data entry in this template would results in an automated answer to a property investment valuation problem involving a wide range of property interests. However, with a focus on occupancy rights in Nigeria where ground rent is paid to the government while the holder of the right of occupancy receives commercial property rent from a sitting tenant, the template is adjusted accordingly to align with the content of equation 6.

Notwithstanding the feat in the valuation template in Exhibit 7 above, a simplified alternative to the ARRY valuation template was presented from the first principle using calculations availed by equation 12. The essence is to demystify the calculations in the template as much as possible to the understanding of readers and scholars who might not be very conversant with the style and presentation associated with the ARRY model of property investment valuation.

6.4 Model Testing and Validation Technique

Procedure for model testing and validation include conduct of preliminary statistical analysis of input variables; analysis of the gearing effect posed by the non-revision of ground rents relative to the growth of sub-rent; the use of the valuation template in Exhibit 3 in

conjunction with equation 23 to value the right of occupancy posed in the case study; and the validation of the modified UK real value/short-cut DCF valuation using the full DCF technique alongside the valuation templates in Exhibits 1, 2, and 4. Using Exhibit 4, the equivalent yield calculation and valuation were aimed at determining the growth implicit internal rate of return of the property interest. With recourse to the exercise posed in sub-section 7.1, the modified UK real value/short-cut DCF model is adjudged validated if its valuation appears identical to those produced from the full DCF technique, modified rational model, Crosby's short-cut DCF model, and the equivalent yield valuation model. The inclusion of the ARRY model in this article is to avail valuers and valuation scholars with the latest alternative model from the Australasian/North American pedagogy aimed at enhancing the formation of market value opinion. Using an Excel® spreadsheet, the calculation of investment functions and income multipliers were expressed in four decimal places, while valuations were concluded with accuracy to the nearest whole number of the Nigerian currency, the Naira (₦).

■7.0 MODEL TESTING AND VALIDATION

This section examines the application of the five contemporary valuation models to a hypothetical commercial property situated on a land in a local government area (municipal council) in Nigeria which had been secured by a grant of statutory right of occupancy and where revision of ground rent has been waived by virtue of sections 5(1)(g), 17(1) and 17(2) of the Land Use Act since the grant of the occupancy right. Actual data pertaining to the local property market and the general economy of Nigeria were deployed for the valuation exercise.

7.1 Leasehold Valuation Case Study

The hypothetical valuation pertains to a commercial property situated at Anyigba, a settlement in Okura district of Dekina Local Government Area of Kogi State in Nigeria. The property comprises 15 units of shops built on a 3,600 square metres parcel of land allocated via statutory right of occupancy with an unexpired term of 50 years as at 14th July, 2020. The market rent of each shop measuring about 9.5 square metres of lettable space is ₦100,000 per annum (subject to 2 yearly upward reviews). Extant Legislation in Kogi State indicates a ground rent of ₦55/m^2 per annum for commercial properties (Kogi State Government of Nigeria, 2017). If it is assumed that the ground rent remains fixed for the unexpired term of occupancy right without any foreseeable revision by the Kogi state governor with respect to urban and non-urban lands in accordance with Sections 5(1)(d), 5(1)(g), and 17 of the Land Use Act as observed across Nigeria (Onuoha et al., 2015), then the gearing effect of such interest shall warrant the use of equation 23 or its variants to carry out such valuations. The valuation date is 14 July 2020, while the rental payments are assumed paid annually in arrears to be consistent across all models.

Table 1 contains some of the data used to support the valuation to include indirect rental evidence of lettable shops and initial yields, the maximum coupon rates on Federal Government of Nigeria Bonds from 2011 to 2020 and average inflation rates from 2011 to 2020. Based on the author's knowledge of the local property market, the rationale behind limiting the initial yield data to the 10-year period was attributed to the dearth of active sales of commercial properties prior to the year 2011. Also based on the knowledge of the local property market, annual ground rents and sublease rental payments are made at the beginning of the period (BOP) but for the purpose of the valuation models in this article, which are designed to sustain academic debate on the subject matter, the treatment of the valuation has been limited to end of the year payments. In each instance however, the valuation of cash flows in arrears (EOP) can be adjusted to valuation of cash flows in advance (BOP) by multiplying the ensuing EOP valuation by the sum of unity and the appropriate discount rate.

Table 1 Calculation of implied rental growth rate, risk-free yield and risk premium

Year	Market rent per m^2 per annum [a.] (₦)	Index of rent	Rental value change (%)	Average Inflation rate [b.] (%)	Maximum FGN Bond yield [c.] (%)	Initial yield [a.] (ARY) (%)	Yield gap	Risk premium (%)
2010	5,790	100.00	-	-	-	-	-	-
2011	6,106	105.46	5.46	10.85	16.5	8.15	8.35	-2.89
2012	6,369	110.00	4.31	12.24	13.5	8.16	5.34	-1.03
2013	6,843	118.19	7.44	8.52	13.8	8.24	5.56	1.89
2014	7,369	127.27	7.69	8.05	13.6	8.52	5.08	2.61
2015	7,895	136.36	7.14	9.01	16.99	8.24	8.75	-1.61
2016	8,421	145.44	6.66	15.63	16.43	8.51	7.92	-1.26
2017	8,948	154.54	6.26	16.55	16.99	8.30	8.69	-2.43
2018	9,474	163.63	5.88	12.14	15.83	8.00	7.83	-1.95
2019	10,000	172.71	5.55	11.39	14.64	8.27	6.37	-0.82
2020	10,527	181.81	5.27	12.26	12.98	8.51	4.47	0.80
Geometric mean			**6.16**	**11.63**	**15.12**	**8.29**		**-0.69**

Data sources

a. WhatsApp platform of the Members of the Kogi State Branch of the Nigerian Institution of Estate Surveyors and Valuers (NIESV), Mitula Homes (https://homes.mitula.com.ng/commercials-kogi) and Nigeria Property Centre (https://nigeriapropertycentre.com/for-sale/kogi?q=for-sale+kogi)

b. Central Bank of Nigeria (2020b)

c. Central Bank of Nigeria (2020a)

With recourse to equation 24, market rental data were used to compute the index of rent as follows:

$$\text{Index of rental value} = \frac{\text{Current market rent}}{\text{2010 market rent}} \times 100 \quad (24)$$

Thereafter, the annual rental value changes in Table 1 were calculated using equation 25:

$$\text{Annual rental value change, RV}\Delta \text{ p.a.} = \frac{\text{Current index}}{\text{preceding index}} \times 100 \quad (25)$$

The annual yield gap from 2011 to 2020 was calculated using equation 26; where r_0 = bond yield or the risk-free yield and k_i = initial yield. This is in consonance with the seminal works of Adams et al. (2003); Baum and MacGregor (1992); Baum (1988); Crosby et al. (2016); Jones et al. (2015) and Wyatt (2013) concerning simple determination of expected risk premium from property investments:

$$\text{Yield gap p.a.} = r_0 - k_i \quad (26)$$

So that annual risk premiums, r_{pa} were calculated using equation 27 or 28:

$$\text{Risk premium, } r_{pa} = RV\Delta \text{ p.a.} - \text{Yield gap p.a.} \quad (27)$$
$$\text{Risk premium, } r_{pa} = RV\Delta \text{ p.a.} - r_0 - k_i \quad (28)$$

Where n = 10 and x_1 to x_n represent the annual values of each assessed variable, the geometric means of rental value change, average inflation rate, bond yields, initial yield and risk premium from 2011 to 2020 were calculated using equation 30 as simplified from equation 29:

$$GM(\mathbf{X}) = \left(\left(\prod x_i \right)^{1/n} - 1 \right) \times 100 \quad (29)$$

$$GM(\mathbf{X}) = \left(\sqrt[n]{(1+x_1)(1+x_2)(1+x_3)(1+x_4)(1+x_5)\ldots(1+x_n)} - 1 \right) \times 100 \quad (30)$$

While 12.26% inflation rate was recorded in Nigeria as at March 2020, a cumulative average inflation rate of 11.63% was recorded between the year 2011 and 2020. The 11.63% cumulative average inflation rate surpasses the geometric mean of rental value change over the 10-year period. This valuation case study deployed the 6.16% geometric mean of rental value change over the 10 year period to represent the implied rental growth rate per annum in consonance with a similar study by Udoekanem et al. (2014). Table 1 indicates that the passing rent for this class of commercial property during the 10-year period capitalizes at an average of 8.29%. The evidence of sales used to determine these capitalization rates are predominantly rights of occupancies granted for the purpose of commercial developments. Further applicable to this class of commercial property include 15.12% risk-free rate (r_0) and a risk premium, r_p of -0.69% (Table 1). With recourse to equation 31, the nominal yield for the subject property was determined as follows:

$$\text{Equated yield, } e = r_0 + r_p \quad (31)$$
$$\text{Equated yield, } e = 15.12\% + (-0.69\%) = 15.12\% - 0.69\%$$
$$\text{Equated yield, } e = 14.43\%$$

Preliminary calculations:

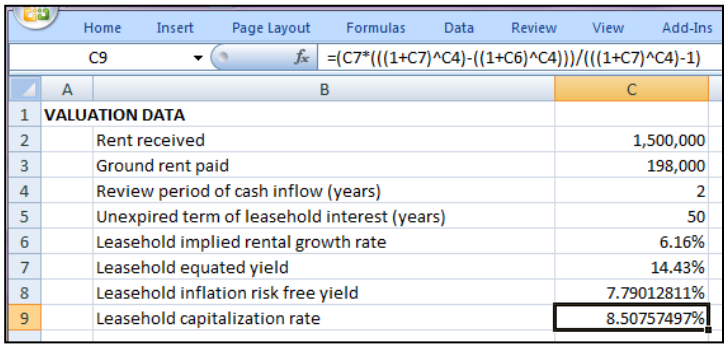
Market rent of a unit of shop = ₦100,000 p.a.
Total number of shops = 15 units
Total market rent realizable = 15 units × ₦100,000 p.a. = ₦1,500,000 p.a.
Area of parcel allocated = 3,600 m^2
Index of ground rent payable = ₦55/m^2
Actual ground rent payable = 3,600 m^2 × ₦55/m^2 = ₦198,000 p.a.

7.2 Data Specification

Valuation data for this case study were arranged in Excel® spreadsheet as shown in Figure 2. The inflation risk free yield, i was calculated using equation 15. Furthermore, the leasehold all risks yield, k_o as indicated using the cursor on cell C9 was calculated using equation 32 where all the variables retain their original meaning, while t = rent review period of 2 years:

$$k_o = e - e\left(\frac{(1+g)^t - 1}{(1+e)^t - 1} \right) \quad (32)$$

Besides all the other parameters for this valuation that were directly imputed in cells C2 to C7, the inflation risk free yield and leasehold cap rate were calculated with reference to the imputed cells C4, C6, and C7 respectively. To avoid redundancy, all the valuations conducted within this spreadsheet were referenced to the data imputed and calculated across cells C2 to C9 as indicated in Figure 2, except for the ARRY valuation model in Figure 9 where a distinct worksheet was deployed, yet retaining most fundamental inputs.



	A	B	C
1	VALUATION DATA		
2		Rent received	1,500,000
3		Ground rent paid	198,000
4		Review period of cash inflow (years)	2
5		Unexpired term of leasehold interest (years)	50
6		Leasehold implied rental growth rate	6.16%
7		Leasehold equated yield	14.43%
8		Leasehold inflation risk free yield	7.79012811%
9		Leasehold capitalization rate	8.50757497%

Figure 2 Data for the valuation of geared leasehold profit rent

7.3 Identifying Profit Rent Gearing

In tandem with Baum and Yu (1985a), growth rates, G_P and index of growth rates G_{Index} for profit rent were calculated using equations 33 and 34 respectively.

$$G_P = \left\{\sqrt[3]{\left(PR_t / PR_{t-3}\right)} - 1\right\} \tag{33}$$

$$G_{index} = \left\{\left(\sqrt[t-2]{\left(1 + \left(PR_t - PR_0\right)\right)/PR_0}\right) - 1\right\} \tag{34}$$

For equations 33 and 34, the value of t = value in the second column of an observed tranche in Table 2, PR_{t-2} = profit rent in each preceding tranche, PR_t = profit rent at the end of a tranche, PR_0 = profit rent in the base tranche - "1 - 2 years".

For evidence of the gearing effect in Table 2, ground rent for the 1st tranche represents 13.2% of the sub-rent and steadily diminishes to about 0.75% as the occupancy right approaches the expiry date. The growth index of profit rent in the first tranche was identical to profit rent growth rate of 7.07% recorded in the same period. As both *Gp* and G_{index} continue to decline towards the end of the 50-year term, the profit rent growth rate stabilized at about 6.22% between the 46th to 48th year. For the gearing effect in Figure 3, profit rent continues to represent at most 2% of the gross sub-rent until the 32nd year.

Table 2 The gearing effect of the leasehold profit rent in the valuation case study

End of Year	End of year, t	Sub-rent (₦)	Head rent (₦)	Profit rent (₦)	Growth rate of profit rent (G_P)	Profit rent growth index (G_{Index})
1 - 2	2	1,500,000	198,000	1,302,000	-	-
3 - 4	4	1,690,492	198,000	1,492,492	0.0707	0.0707
5 - 6	6	1,905,175	198,000	1,707,175	0.0695	0.0701
7 - 8	8	2,147,122	198,000	1,949,122	0.0685	0.0696
9 - 10	10	2,419,795	198,000	2,221,795	0.0677	0.0691
11 - 12	12	2,727,096	198,000	2,529,096	0.0669	0.0686
13 - 14	14	3,073,422	198,000	2,875,422	0.0663	0.0683
15 - 16	16	3,463,730	198,000	3,265,730	0.0657	0.0679
17 - 18	18	3,903,605	198,000	3,705,605	0.0652	0.0676
19 - 20	20	4,399,341	198,000	4,201,341	0.0648	0.0672
21 - 22	22	4,958,033	198,000	4,760,033	0.0644	0.0670
23 - 24	24	5,587,677	198,000	5,389,677	0.0641	0.0667
25 - 26	26	6,297,281	198,000	6,099,281	0.0638	0.0665
27 - 28	28	7,097,002	198,000	6,899,002	0.0635	0.0662
29 - 30	30	7,998,282	198,000	7,800,282	0.0633	0.0660
31 - 32	32	9,014,021	198,000	8,816,021	0.0631	0.0658
33 - 34	34	10,158,752	198,000	9,960,752	0.0629	0.0657
35 - 36	36	11,448,859	198,000	11,250,859	0.0628	0.0655
37 - 38	38	12,902,801	198,000	12,704,801	0.0627	0.0653
39 - 40	40	14,541,387	198,000	14,343,387	0.0625	0.0652
41 - 42	42	16,388,064	198,000	16,190,064	0.0624	0.0650
43 - 44	44	18,469,259	198,000	18,271,259	0.0623	0.0649
45 - 46	46	20,814,754	198,000	20,616,754	0.0622	0.0648
47 - 48	48	23,458,115	198,000	23,260,115	0.0622	0.0647
49 - 50	50	26,437,168	198,000	26,239,168	0.0621	0.0646

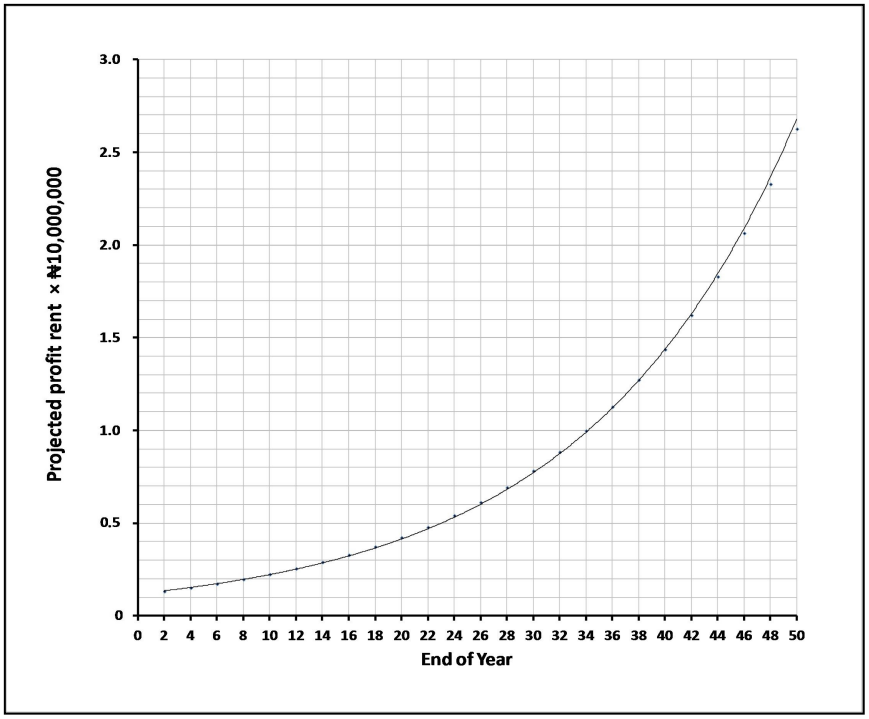


Figure 3 Gearing leasehold profit rent gearing

After the 32nd year, the impact of the ground rent on profit rent determination diminishes rapidly; so that a large proportion of the profit rent can be attributed to the sub-rent. If the sub-rent and ground rent in Table 2 were used to produce further forecasts of profit rents to say about 90 years, the values of G_p would approach approximately 6.16%; while the value of G_{index} would revolve around 6.31% up to the 99th year of the forecast.

7.4 Application of the Modified UK Real Value/Short-Cut DCF Model

Figure 4 features the valuation of the geared leasehold interest (occupancy right) using the modified UK real value/short-cut model developed in this article. In this model, the calculation of income multiplier for rent received occurred in two stages. The first was the product of the compound interest on ₦1 in a 50-year term at 7.79012811% inflation risks free yield and the PV of ₦1 in 50 years at 7.79012811% inflation risk free yield. Multiplying the ensuing result by the Years Purchase [YP] in perpetuity at 8.50757497% all risks yield would reduce the Years Purchase to 11.4780, being an equivalent income multiplier for leasehold investments.

Home Insert Page Layout Formulas Data Review View Add-Ins Acrobat Monte Carlo

D17 =(((1+C7)^C4)-1)/(C7*(((1+C7)^C4)-((1+C6)^C4)))

	A	B	C	D	E	F
15					₦	₦
16		Rent received			1,500,000	
17		Y.P. in Perpetuity @ 8.50757497%		11.7542		
18		Compound interest on ₦1 in 50 years @ 7.79012811%	41.5549			
19		P.V. of ₦1 in 50 years @ 7.79012811%	0.0235	0.9765	11.4780	17,217,026
20	Less	Rent paid			198,000	
21		Y.P. for 50 years @ 14.43%			6.9218	1,370,518
22		Capital value of leasehold interest				15,846,508

Figure 4 Valuation using the modified UK real value/short-cut DCF model [Equation 22]

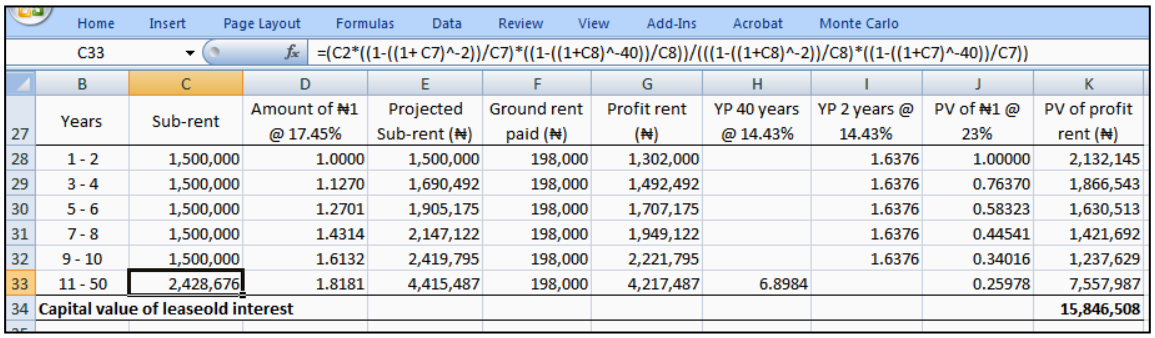
With reference to data in Figure 2, the position of the cursor on cell D17 in Figure 4 indicates in the formula bar the spreadsheet version of the equation for YP in perpetuity at the 8.50757497% all risks yield. The fixed ground rent of ₦198,000 was multiplied by the YP for 50 years at a 14.43% nominal yield to obtain a discounted cash flow of ₦1,327,518, which when subtracted from the capital value of sub-rent amounts to a leasehold capital value of ₦15,846,508. The next sub-section details the validation of the modified UK real value/short-cut DCF model.

7.5 Model Validation Using Variants of Contemporary UK Leasehold Valuation Models

This section reports the outcome of the validation process of the modified UK real value/short-cut DCF model developed in this article using an array of contemporary investment valuation models for leasehold interest commencing with the full DCF valuation technique and ending with the conventional equivalent yield valuation model.

7.5.1 The Full Explicit DCF Valuation Model

The full DCF valuation in Figure 5 was prepared according to the procedure advocated by Sayce et al. (2006) except that the terminal sub-rent covering the period of 11 to 50 years was adjusted using equation 1 as indicated in the formula bar where the cursor was placed in cell C33. The application of equation 2 would yield similar result for the sub-rent to an adjusted value of ₦2,428,676 for the remaining 40-year period. This procedure was deigned to moderate the length of the DCF valuation.



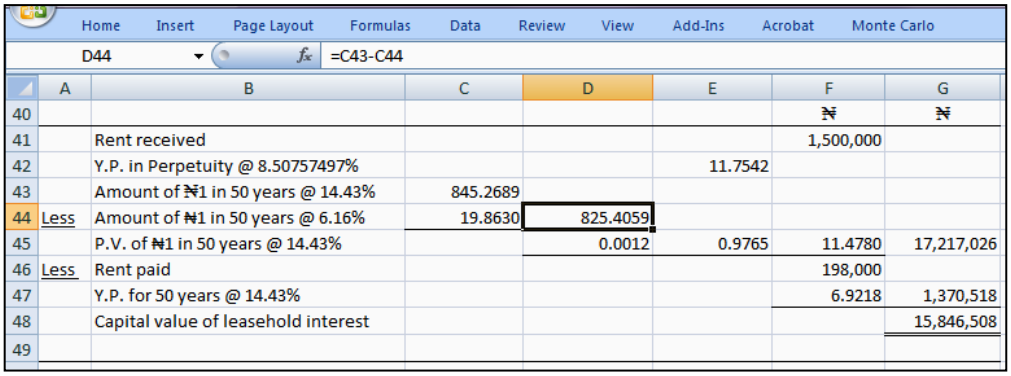
	B	C	D	E	F	G	H	I	J	K
27	Years	Sub-rent	Amount of ₦1 @ 17.45%	Projected Sub-rent (₦)	Ground rent paid (₦)	Profit rent (₦)	YP 40 years @ 14.43%	YP 2 years @ 14.43%	PV of ₦1 @ 23%	PV of profit rent (₦)
28	1 - 2	1,500,000	1.0000	1,500,000	198,000	1,302,000		1.6376	1.00000	2,132,145
29	3 - 4	1,500,000	1.1270	1,690,492	198,000	1,492,492		1.6376	0.76370	1,866,543
30	5 - 6	1,500,000	1.2701	1,905,175	198,000	1,707,175		1.6376	0.58323	1,630,513
31	7 - 8	1,500,000	1.4314	2,147,122	198,000	1,949,122		1.6376	0.44541	1,421,692
32	9 - 10	1,500,000	1.6132	2,419,795	198,000	2,221,795		1.6376	0.34016	1,237,629
33	11 - 50	2,428,676	1.8181	4,415,487	198,000	4,217,487	6.8984		0.25978	7,557,987
34	**Capital value of leaseold interest**									**15,846,508**

Figure 5 The full DCF valuation of the geared leasehold profit rent

The full explicit DCF valuation in Figure 5 retuned a capital value of ₦15,846,508 to further validate the valuation churned out by the modified UK real value/short-cut DCF model developed in this article.

7.5.2 The Modified Rational Model

Figure 6 features valuation using a variant of the modified rational model proposed by Baum and Yu (1985b). The calculation of income multiplier for rent received is structured in three phases. First, the difference between the Amount of ₦1 in 50 years at 14.43% equated yield and 6.16% implied rental growth rates was found to be 825.4059. Multiplying this index by the PV of ₦1 in 50 years at 14.43% equated yield would reduce the Years Purchase in perpetuity at 8.50757497% all risks yield to 11.4780, being an equivalent income multiplier for the investments.



	A	B	C	D	E	F	G
40						₦	₦
41		Rent received				1,500,000	
42		Y.P. in Perpetuity @ 8.50757497%			11.7542		
43		Amount of ₦1 in 50 years @ 14.43%	845.2689				
44	Less	Amount of ₦1 in 50 years @ 6.16%	19.8630	825.4059			
45		P.V. of ₦1 in 50 years @ 14.43%		0.0012	0.9765	11.4780	17,217,026
46	Less	Rent paid				198,000	
47		Y.P. for 50 years @ 14.43%				6.9218	1,370,518
48		Capital value of leasehold interest					15,846,508
49							

Figure 6 Valuation using the modified rational model [Equation 3]

Just like the modified UK real value/short-cut DCF valuation in Figure 4 and the full DCF valuation in Figure 5, a capital value of ₦15,846,508 was equally returned from the modified rational model in Figure 6. When compared to the modified UK real value/short-cut DCF valuation in Figure 4, the four data required to calculate the income multiplier for cash inflows in this variant of modified rational model include the 8.50757497% all risks yield, 14.43% equated yield, 6.16% implied rental growth rate, and the 50 years unexpired term. This model implicitly addresses the 2-yearly rent review period, and the inflation risk free yield by using their analytical surrogates.

7.5.3 Crosby's Real Value/Short-Cut DCF Model

With reference to the data in Figure 2, the cursor on cell C56 in Figure 7 indicates the spreadsheet equivalent of the 3-in-1Years purchase formula for the valuation of the sub-rent. This income multiplier for rent received (3-YPs formula) requires four input parameters comprising the 2-yearly rent review period, 14.43% leasehold equated yield, 7.79012811% inflation risk free yield, and the 50 years unexpired term.

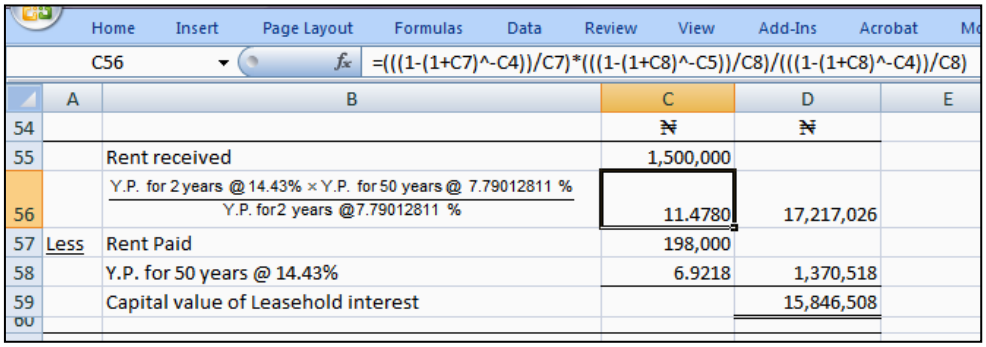


Figure 7 Valuation using the Crosby's real value/short-cut DCF model [Equation 4]

In Figure 7, the Crosby's real value/short-cut DCF valuation model returned the capital value of the subject property in the sum of ₦15,846,508 which is similar to the valuation churned out by the modified UK real value/short-cut DCF model developed in this article. This valuation also appears identical to those produced by the full DCF technique and the modified rational model respectively. Just as in Figure 4, the 3-YPs formula in Figure 7 capitalized the ₦1,500,000 sub-rent to arrive at capital value of cash inflow in the sum of ₦17,217,026. Subtracting ₦1,370,518 representing the capital value of ground rent paid from this figure would amount to a capital value in the sum of ₦15,846,508, which reconciles with the preceding valuations.

7.5.4 Equivalent Yield Calculation and Valuation

Figure 8 features the calculation of equivalent yield and the valuation associated with the same equivalent yield. With recourse to capital value of interest in the leasehold property to the tune of ₦15,846,508 drawn from valuations in Figure 4 to 7, the conduct of scenario analysis on the initial appraisal data in consonance with the seminal works of Baum et al. (2014) returned 8.04472873% equivalent yield.

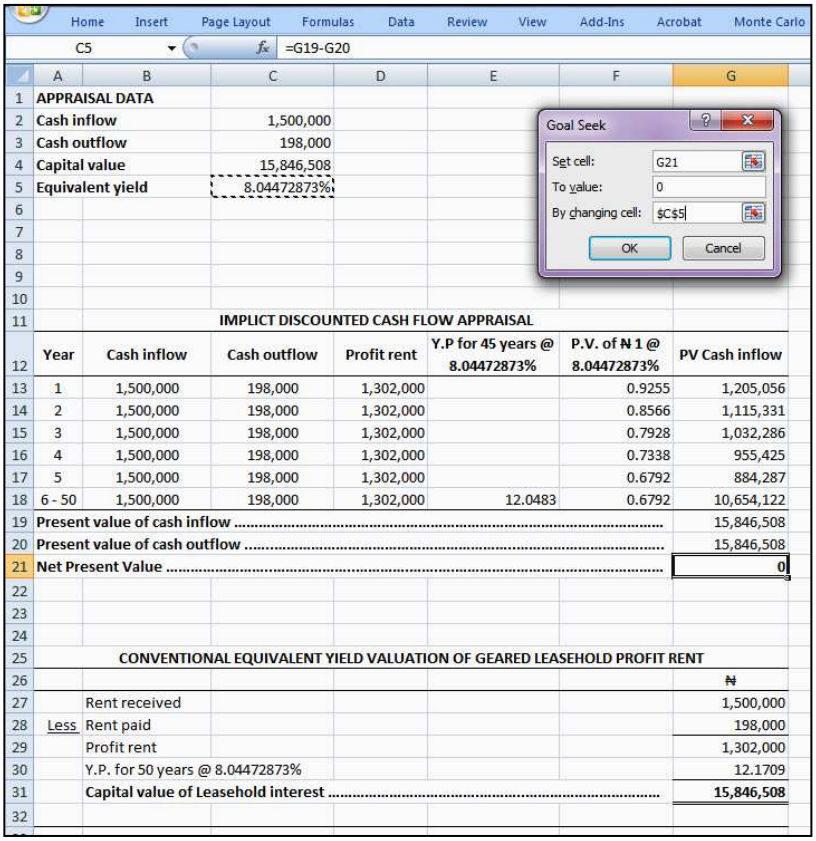


Figure 8 Conventional leasehold equivalent yield determination and valuation

From the extant perspective of explicit DCF appraisal, this equivalent yield is less than the all risks yield by 0.84%. Using this equivalent yield to conventionally value the geared leasehold would return capital value in the sum of ₦15,846,508. The valuation in Figure 8 appears reliable given the identical valuations in Figures 4 to 7 for the 50-year unexpired interest in the occupancy right. The rationale for the reconciliation of these four valuations in Figures 4 to 7 is the underlying relationships among the input variables of the respective valuation models.

7.6 Application of the ARRY Valuation Model

As noted in sections 5 and 6 of this article, the ARRY valuation for the geared leasehold profit rent presented is a departure from the explicit DCF and the UK real value models. For the purpose of the valuation case study posed in section 7.1 and equations 7 and 13, the input data for the ARRY model is presented in Table 3.

Table 3 Data deployed in the ARRY model of property investment valuation

Parameter	Value/Description
Rent received, C_C	₦1,500,000.00
Ground rent paid, C_o	₦198,000.00
Number of years to the next upward review of sub-rent,	0 years
Review frequency for cash inflow, F	2 years
Unexpired term of leasehold interest, T	50 years
Expected (nominal) inflation rate, I_e (See Table 1)	11.63%
Leasehold nominal growth rate, $Go = (I_e + G_r)$ (See Table 1)	6.16%
Real growth rate of cash flow, $G_r = (Go - I_e)$	-5.47%
All risk real yield of cash inflow, Y_A	8.27%
Overall required nominal yield rate, $Yo = (Y_A + I_e + G_r)$	14.43%
ARRY capitalisation rate of cash inflow, R_A^F (Determined using equation 8)	8.50757497%
ARRY capitalisation rate of cash outflow, R_A^T (Determined using equation 9)	14.10759855%
Receipt of cash inflow and cash outflow	End of the year (In arrears)

With recourse to the valuation data in Table 3, three approaches were deployed in pursuit of the ARRY valuation of the subject property. The first and second approaches entailed the use of the Excel template provided in Jefferies (2018b). Although the two Excel® templates were originally configured to display valuations in Dollars, an adaptation was made in the template used in this study to show the valuation in Naira symbol (₦), being the Nigerian currency. The third approach is the manual deployment of equation 12, being a surrogate ARRY model that strictly applies to the subject property being valued. The ARRY valuations across these three approaches are expected to exhibited minimal or zero deviations.

7.6.1 ARRY valuation using a portfolio template to mimic the term and reversion

The 50-year sublease presented in this case study was valued using a portfolio template to mimic the term and reversion components of the ARRY model. The three vital screenshots extracted for this purpose have been captured in this section. The cash inflow and cash outflows have been treated as annuities in arrears, with the sub-rentals valued on one sheet and present real value (PRV) of ground rental expense as a negative valuation on another sheet, combined into an overall summary in the third sheet as follows:

TERM(S) TO RUN & REVERSIONARY calculations – components of total present real value (PRV):		
PRV of term to run to next rental review (if any) or to lease expiry; or PRV of first review term after initial vacancy (if allowed):		
Current review rental ₦1500000 p.a. with expected real growth over 2 years rent reviews capitalised in perpetuity @ RA2: 8.50757% p.a.:	₦	17,631,346
No vacancy allowed and/or no unexpired rent-free period - capitalised contract rental in perpetuity - as above:	₦	17,631,346
LESS PRV of capitalised contract rental in 2 years time discounted @ YA: 8.27 % p.a.	-₦	15,040,740
PRV of contract rental to run to next rent review in 2 years time:	₦	*2,590,606*
PRV of contract rental(s) from next rent review (If applicable) to current lease expiry:		
Current review rental ₦1500000 p.a. with expected real growth over 2 years rent reviews capitalised in perpetuity @ RA2: 8.50757% p.a.:	₦	17,631,346
PRV of capitalised current review rental real value above, deferred to next rent review in 2 years time discounted @ YA: 8.27 % p.a. :	₦	15,040,740
LESS - PRV of capitalised current review rental real value above, deferred to lease expiry in 50 years time discounted @ YA: 8.27 % p.a. :	-₦	331,802
PRV of rental term from rental review in 2 years to lease expiry in 50 years (by deduction):	₦	*14,708,938*
Current market value (rounded) *CMV* **:**	₦	17,300,000

Figure 9(a) First term of the ARRY valuation of sub-leased property (Sublease rental income)

TERM(S) TO RUN & REVERSIONARY calculations – components of total present real value (PRV):		
PRV of term to run to next rental review (if any) or to lease expiry; or PRV of first review term after initial vacancy (if allowed):		
Contract rental: ₦198000 p.a. with expected real growth over 50 years term to run, capitalised in perpetuity @ RA50: 14.1076% p.a.:	₦	1,403,499
No vacancy allowed and/or no unexpired rent-free period - capitalised contract rental in perpetuity - as above:	- ₦	1,403,499
LESS PRV of capitalised contract rental in 50 years time discounted @ YA: 8.27 % p.a.	₦	26,412
PRV of contract rental to run to next rent review in 50 years time:	**- ₦**	***1,377,087***

Figure 9(b) Second term of the ARRY valuation of sub-leased property (Negative valuation of ground rent)

SUMMARY of term & reversion valuations: components of present real value (PRV):						
PRV of current contract rental over term to run to next rent review:	₦	*1,213,520*	-₦	*1,377,087*	₦	*2,590,606*
PRV of contract rental(s) from next rent review to current lease expiry:	₦	*14,708,938*	₦	-	₦	*14,708,938*
PRV of contract rental(s) from lease expiry to final expiry of renewals:	₦	-	₦	-	₦	-
PRV at rent review (if perpetually renewable); or re-leasing at final expiry:	₦	-	₦	-	₦	-
PRV of real redevelopment value at lease termination:	₦	-	₦	-	₦	-
PRV of unrecovered OPEX: if applicable - see individual property sheets for details	₦	-	₦	-	₦	-
PRV of initial vacancies and/or vacancies on re-leasing:	₦	-	₦	-	₦	-
PRV of CAPEX, added-value (e.g. vacant land) or other adjustments	₦	-	₦	-	₦	-
TOTAL PRESENT REAL VALUES of terms and reversions ± other adjustments	₦	*15,922,458*	-₦	*1,377,087*	₦	*17,299,545*
Current real market value - CMV: (rounded)	₦	15,923,000	-₦	1,377,000	₦	17,300,000
Net initial yield - after any initial vacancies (if applicable) ÷ CMV:		8.1769% p.a.		14.3791% p.a.		8.6705% p.a.
Ex manuscript: Modified UK real value/short-cut DCF valuation model	₦	*15,846,508*				
Difference:	₦	75,950	= Difference of + 0.47928%			

Figure 9(c) Summary of the ARRY valuation of sub-leased property (Valuation and value comparison)

With respect to the subject property, the first approach to the ARRY valuation in Figures 9(a) to 9(c) indicates a capital market value of ₦15,922,458, which is very close to and immaterially different from valuations produced by the UK models in sections 7.4 and 7.5 above. In other words, the ARRY valuation exceeded that from the UK model by ₦75,950 (+0.47928%), which is immaterial given the differences in the methods and the higher internal accuracy of the spreadsheet.

7.6.2 ARRY Valuation Using Single Property Template that Treats Ground Rent Payment as Fixed Operating Expense (OPEX)

In the second approach (Figure 10), the subject property was valued using a variant of the ARRY valuation model treating ground rent payment as operating expenses with a zero nominal growth over the 50-year unexpired term. The theory behind this approach has been explained by Jefferies (2017c, pp. 96-98).

TERM(S) TO RUN & REVERSIONARY calculations – components of total present real value (PRV):		
PRV of term to run to next rental review (if any) or to lease expiry; or PRV of first review term after initial vacancy (if allowed):		
Contract rental: ₦1500000 p.a. with expected real growth over 2 years term to run, capitalised in perpetuity @ RA2: 8.50757% p.a. :	₦	17,631,346
No vacancy allowed and/or no unexpired rent-free period - capitalised contract rental in perpetuity - as above:	₦	17,631,346
LESS PRV of capitalised contract rental in 2 years time discounted @ YA: 8.27 % p.a.	-₦	15,040,740
PRV of contract rental to run to next rent review in 2 years time:	**₦**	***2,590,606***
PRV of contract rental(s) from next rent review (If applicable) to current lease expiry:		
Current review rental ₦1500000 p.a. with expected real growth over 2 years rent reviews capitalised in perpetuity @ RA2: 8.50757% p.a. :	₦	17,631,346
PRV of capitalised current review rental real value above, deferred to next rent review in 2 years time discounted @ YA: 8.27 % p.a. :	₦	15,040,740
LESS - PRV of capitalised current review rental real value above, deferred to lease expiry in 50 years time discounted @ YA: 8.27 % p.a. :	-₦	331,802
No PRV of initial vacancy costs allowed - as currently tenanted.	₦	-
No PRV of re-leasing vacancy, as not allowed.	₦	-
PRV: of OPEX ₦-198000 p.a. escalating @ 0% p.a. until lease expiry in 50 years	-₦	*1,346,319*
PRV of expected CAPEX; other liabilities; non-income based added-value (e.g. vacant land); or other adjustments (see note		
TOTAL PRV: term & reversions ± other adjustments = *CMV*	**₦**	***15,953,225***
Current market value (rounded) *CMV* :	**₦**	**15,953,000**

Figure 10 The second approach to the ARRY valuation of sub-leased property

Unlike the first approach, the advantage of the second approach is that the valuation is presented in a single Excel Sheet. Figure 10 indicates a valuation of ₦15,953,225, which exceeded those produced from the UK models in sections 7.4 and 7.5 by ₦106,717 or 0.67344% (rounded to 5 decimal points). Notwithstanding, the ARRY valuation in Figure 10 is still quite close to and immaterially different from those produced from the UK models including the modified real value/short-cut DCF model developed in this article mainly due to the different methods and higher internal accuracy of the spreadsheet.

7.6.3 ARRY Valuation Using the Simplified ARRY Model for the Subject Property

It could be recalled that equation 13 is a simplified modification of the ARRY valuation model for the subject property. In order to validate this variant of ARRY model, the calculation of the capital market value (CMV) was carried out from the first principle using mathematical approach and applicable data in Table 3. Hence, the CMV is expressed as follows:

$$₦15{,}922{,}458 \Leftarrow \left(\frac{1{,}500{,}000}{0.08507574\ 97} - \frac{198{,}000}{0.14107598\ 55}\right)\left(1-(1.0827)^{-50}\right)$$

In essence, the simplification of the ARRY model in equation 12 reconciles the valuation in section 7.6.3 with that in section 7.6.1, so that the ₦75,950 (+0.47928%) difference between the ARRY valuation and valuations from the UK models remains immaterial. While the simplification of the ARRY model in equation 12 does not in any way relegate the quest for automated valuations in the 21st century, it provides an alternative approach to value computation using handheld calculators and smart phones where computers cannot be readily powered up to produce valuations in spreadsheets especially in Nigeria where inhabitants including practicing valuers have to bear the daunting cost of providing alternative source of electricity to charge laptops and power-up computers in the event of erratic power supply from the national grid. In order to surmount this kind of limitation, Jefferies (2017c) advocated the development of full Excel template of ARRY valuation model in the form of application programs (Apps) for smart phones and tablet PCs as a further research and development project by other researchers.

A comparison of the extant UK models for the valuation case study in Figures 4 to 7 with the valuation of the same sub-leased property using the ARRY model in sub-sections 7.6.1 to 7.6.3 might lead to a value opinion of say ₦15,900,000 for the unexpired interest in the said commercial property. This represents a difference of only ₦53,492 or 0.33642767% from the ₦15,846,508 estimated using the UK valuation models. With respect to sub-section 7.6.1 and 7.6.3, the ARRY model produced an excess valuation of about ₦23,000 on the basis of the ₦15,900,000 value opinion, which represents a difference of only 0.14465%. Given this margin of value differential, it is reasonable to validate the valuation produced by the modified UK real value/short-cut DCF model using the valuation by the ARRY model. These differences are well within an acceptable valuation tolerance, and indeed if the UK models' inputs, calculation and outputs before rounding were shown, then it is probable that the same "accurate" valuations would result, validating all method against each other.

■8.0 DISCUSSION OF RESULTS

The Food and Agriculture Organization of the United Nations (FAO, 2017) advocates the placement of valuations within the context of land policy, which is construed in this article as requiring the restructure of existing valuation models to align with land policy documents. In the case of nationalized land policy, it has been shown that a situation whereby land trustees freeze ground rents in accordance with the provisions of an extant legislation may give rise to a situation synonymous to that in the valuation case study where the 2-yearly review sub-rent grows throughout the term of the grant to the extent that a considerable proportion of the profit rent is attributable to the increasing amount of sub-rent realized at each review epoch indicated in Table 2 and Figure 3 respectively. Providing an evidence of gearing effect for the valuation case study in Table 2 is the manifestation of profit rent as 13.2% of the sub-rent in the 1st tranche which steadily declines to about 0.75% of the revised sub-rent in the last tranche. Hence, the diminishing impact of the fixed ground rent was observed as the occupancy right (lease) approaches expiry date for the hypothetical valuation case study. This result is in tandem with the seminal works of Baum and Yu (1985b) and Baum and Crosby (2008) pertaining to the gearing effect of fixed ground rents; except that a further forecast of profit rents to say about 90 years would approximately equate the growth rate of profit rent with the 6.16% implied growth rate of sub-rent, while the index of the profit rent growth shall revolve around 6.31% up to the 99th year.

The modified real value/short-cut DCF valuation model was successfully developed in this article by blending the inputs of the modified rational model with that of the Crosby's real value/short-cut DCF model leading to model surrogates that still featured the equated yield, all risks yield, inflation risk free yield, and the unexpired term of the lease while remaining implicit about rental growth rate and/or the rent review period in a structure. The model requires the same inputs as the original Crosby's real value/short-cut DCF model in the area of equated yield and inflation risk free yield on the one hand; while on the other hand, it appears similar to the rational model in the sense that the YP in perpetuity is discounted to equivalent income multiplier for terminal interests.

Although the UK explicit DCF and real value models were developed during the era of manual hand calculations, the deployment of formula-audited spreadsheets and database-oriented programmes in recent times has simplified their application across various dimensions of timing for the receipt and payment of cash flows. It should be appreciated that this feat is not an exclusive preserve of the ARRY valuation model because even the UK models have been produced in commercial software packages like KEL® (Baum et al., 2014), which is configured to handle property investment valuations in that clime. Among other software packages used over the years include Dyna®, Circle®, Cougar® (Parker & Robinson, 2002), and Argus® (Fisher & Louziotis, 2013; Parker & Robinson, 2002), which may be available as DCF appraisal software with interoperable capabilities in MS Excel® (see ARGUS Software, 2014).

Notwithstanding, the valuation of geared leaseholds involving the modified UK real value/short-cut DCF model in this article was undertaken by customizing formula-audited spreadsheets in the conventional term and reversion format understood by valuers of the Nigerian and UK extraction. The associated formula for the modified UK real value/short-cut DCF model in equation 22 could also be manually deployed to value similar types of interests in Nigeria.

In a related development, the valuation of similar geared leaseholds using the ARRY valuation model was approached using the customized ARRY valuation template in MS Excel®. Prior to this, the associated ARRY model was compressed into equation 12 so that comparisons were made between the ARRY valuation churned out from the Excel® template and that computed manually (see Section 7.6.3).

With recourse to the hypothetical valuation case study, the modified UK real value/short-cut DCF model returned valuation that appear identical to those from the full DCF, modified rational model, and the original Crosby's real value/short-cut DCF model in the sum of ₦15,846,508 coupled with an equivalent yield of about 8.04472873%. The rationale for the reconciliation of valuations across these models is the strict synergy among their inputs variables as confirmed in similar studies (Baum & Yu, 1985b; Crosby, 1996; Crosby et al., 1997; Jefferies, 2009b). However, Jefferies (2017b) did not perceive the UK real value models as being truly "real value" models, stressing that these UK models and their successors (including the modified UK real value/short-cut DCF model developed in this article) might likely suffer the same fate. In the place of these UK models, Jefferies (2010) had proffered the ARRY model, which according to him would gain acceptance among the fraternity of valuers and appraisers following widespread promotion and conduct of empirical testing of the model.

The valuation of similar geared leaseholds using the ARRY valuation model was approached using two scenarios of customized ARRY valuation templates in MS Excel®. Prior to this, the model pertaining to this valuation was compressed into equation 13, so that comparison could be made between the ARRY valuation churned out from the Excel template and that computed manually (See Section 7.6.3). The manual computation involving equation 12 and the ARRY portfolio valuation template returned capital market value of the subject property in the sum of ₦15,922,458, while the ARRY single property valuation template designed to treat ground rent payment as fixed operating expense (OPEX) returned capital market value of the same property in the sum of ₦15,953,225. Surprisingly, the ARRY valuation involving equation 12 (sub-section 7.6.3) and the ARRY portfolio valuation template (sub-section 7.6.1) returned identical valuations notwithstanding that the former was performed manually, while the latter was automated in a spreadsheet. Contributing to the identical valuations is the higher internal accuracy of the spreadsheet with screenshot in Figures 9(a) to 9(c), and the use of 10-decimal place input data in support of the manual calculation in sub-section 7.6.3 using a handheld electronic calculator.

Jefferies (2010) outlined the three benchmarks for the empirical test and acceptance of ARRY models to include equivalence, superior accuracy, and reliability. In terms of reliability, all the UK models including the modified real value/short-cut DCF model exhibited internal consistency following the identical valuations produced. Contributing to this is the inter-relationship among their input parameters. On the other hand, the ARRY model only showed reliable results following the comparison of the manual computed ARRY valuation using equation 12 and the ARRY portfolio valuation template in Excel®. When compared to the ARRY model, the modified UK real value/short-cut DCF model produced reliable valuations with less than 1% margin of variation. Contrary to these, the question of accuracy depends on the tool of value computation (spreadsheet automation or manual computation) deployed and how input parameters are handled in the model or value computation tool. For instance, the manual ARRY valuation involving equation 12 and the ARRY portfolio valuation template in Excel® returned identical capital market value owing to the higher internal accuracy of the spreadsheet on the one hand, and the use of 10-decimal place input data in manual calculation (equation 12) on the other hand. Surprisingly, however, the second variant of the ARRY model designed to treat ground rent payment as operating expenses produced valuations that deviated from those produced from equation 12 and the ARRY portfolio valuation template. In spite of this deviation, the ensuing valuation still satisfied the equivalence benchmark.

It can be recalled that all the ARRY valuations exhibit a margin of less than 1% when compared to the valuations from the UK models; including the modified UK real value/short-cut DCF model developed in this article. The less than 1% difference is adjudged to be an acceptable margin of variation across the ARRY and the UK models thereby validating all methods against each other for the purpose of valuing occupancy rights in Nigeria with gearing potentials. Contrary to the ARRY model which entails the calculation of capitalization rates using real yields, the valuation of fixed rent paid in the full DCF, modified rational model, and the two variants of the real value/short-cut DCF models was with recourse to a single nominal yield on the premise that the capitalization rate for non-revisable and/or non-growth incomes remains the nominal rate of interest (Baum & MacGregor, 1992; Ifediora, 2005). The implication is that higher valuations for the subject property shall be produced across the ARRY and the UK models on the basis of beginning-of-period cash flows, so that the margin of variation across these models shall still be immaterial.

■9.0 CONCLUSION

Successfully developed in this study is a modified UK real value/short-cut DCF model for the valuation of head-leased occupancy rights with fixed and non-revisable ground rents pursuant to the provisions of the Nigeria Land Use Act. The model was developed by blending the inputs of the modified rational model with that of the Crosby's real value/short-cut DCF model which appears similar to existing full DCF, modified rational, and Crosby's real value/short-cut DCF models. This study also gained insights into the ARRY model that was developed in New Zealand and further modified it for the purpose of valuing head-leased occupancy rights in Nigeria. Using actual data from a section of the Nigerian property market to value a hypothetical head-leased occupancy right in Nigeria, the modified real value//short-cut DCF model returned valuations identical to those churned out from the full DCF, modified rational model, and Crosby's real value/short-cut DCF model. Just like the extant growth explicit/implicit UK models for property investment valuations, the modified UK real value/short-cut DCF model appears to be applicable to the valuation of rights of occupancy in Nigeria that are characterized by fixed ground rents and revisable sub-rents that are subject to growth over the term of grant. With recourse to a valuation case study in

Nigeria, the capital market value arising from the use of the modified UK real value/short-cut DCF model exhibited minimal deviation from the capital market value of the same property arising from the use of the ARRY model credited to Jefferies (2010).

The modified UK real value/short-cut DCF model for the valuation of geared leasehold profit rent that was developed in this study compares with the ARRY model for the same purpose on the basis of equivalence and reliability. Although further research pertaining to the comparison of ARRY valuation and the UK real value/short-cut DCF valuation formulary for beginning-of-period cash flows has been proffered in this study, it is hypothesized that if both classes of models produced equivalent valuations for end-of-period payments, they might likely do the same if beginning-of-period cash flows were valued.

Given the awareness of the provision of the International valuation Standards regarding the appropriate use of nominal and real rate of returns in the income valuation approach (Jefferies, 2010; Parker, 2016), this study recommends a synergy in the teaching and pedagogy of emerging real value models with extant nominal currency valuation models. Besides its recommended use among valuers in Nigeria, the modified UK real value/short-cut DCF model and its variants are adjudged to be applicable to the valuation of head-leased occupancy rights of the ground leased, which are subject to subleases especially in climes that exercise a nationalized land policy that accords a trustee (state governor) the executive powers to freeze ground rents while sub-rent continues to grow throughout the term of the grant to the extent that the proportion of the fixed ground rent to the sub-rent diminishes considerably as the interest approaches its expiry date.

The minimal variation between valuations produced by the ARRY model and the modified UK real value/short-cut DCF models as applied to the valuation case study in this article points to the possibility that the ARRY model might gain acceptance among Nigerian valuers. However, the timing of such acceptance cannot be actually predicted given the over six decades of lock-in effect arising from the exposure to UK valuation models. Notwithstanding, sellers and buyers seeking value information in Nigeria may exercise preference for specific investment valuation methods that address their motives. For instance, the relatively lower valuations produced by the UK models might favour prospective buyers, while the relatively higher valuations from the ARRY model might be preferred by sellers, so that the outcome of any purchase/sales decision arising from these valuation models shall be anchored on the bargaining power of the parties.

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